The Arithmetic Teacher

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One Classroom, with Arithmetic and Justice for All
JO McKEEBY PHILLIPS

Low Visibility, or, Ideas in a Fog
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CLYDE G. CORLE

THE ARITHMETIC TEACHER

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THE ARITHMETIC TEACHER

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One Classroon, with Arithmetic and Justice for All

Jo McKeeby Phillips

D. C. Heath and Company, Boston

An eminent and articulate British scientist, J. Bronowski, has written an article called "Planning for the Year 2000."[1] Because his predictions agree perfectly with those of other scientists, in our own country and in other countries, and because he is disposed to consider the social implications of the scientific advances in many fields, I should like to call attention to them here.

Bronowski states that the year 2000 will be characterized by a pervading effect of three scientific advances which are already in evidence to a lesser degree: atomic energy, control of our biological environment, and automation. This last item, automation, is already causing wide-spread concern among our labor force, but Bronowski says that in another forty years the unskilled worker will be as extinct as the dodo, and adds, "Today we still distinguish, even among repetitive jobs, between the skilled and the unskilled; but in the year 2000 *all* repetition will be skilled. We simply waste our time if we oppose this change; it is as inevitable as the year 2000 itself—and just as neutral.

"But its implications go very deep. For it will displace the clerk as well as the fitter; and the ability to balance a ledger will have no more value, or social status, than driving a rivet. . . . And this is why these speculations about the year 2000 are in place: because the shift is already going on, and it is our business to foresee now where it is certain to take us."

Surely our schools will be derelict indeed if they fail to plan what they are doing right now in the light of trends which are very plain to members of the scientific and industrial communities. Too often a voice of foresight, a voice of reason, is a voice crying in the wilderness. After the general public and certain strong groups of educators had spent several years ignoring the pleas of the real statesmen in our own profession, events of recent months have resulted in the schools' being given a mandate to produce instant scientists and mathematicians, if possible, retroactively. Shortly after Dr. Killian took his present office as Scientific Advisor to the President, he was asked, "How does one produce a sputnik?" He replied, "You start with a scientist in the second grade; preferably in the first grade." It follows, then, that if we are to produce productive thinkers for today, for tomorrow, and for the evolving social structure in which there is no place for the unskilled worker, we must try to develop the maximum potential of each student in every grade. We must begin at once, using what we already have to work with.

Opportunity in Every Classroom

To develop the maximum potential of each student requires an effective and easily implemented means for dealing with the individual differences which exist among pupils in every class.

It is my intention here to present a few ideas which have proved workable in the type of classroom situation in which most teachers find themselves most of the time. There are more than 30 pupils in the class. The range of measurable academic aptitude is from dull normal to very superior. The pupils come from a wide variety of homes, with environmental factors favorable to education ranging from meager to lush, as the saying goes. The school budget allows for modest amounts and varieties of equipment and supplies, none very expensive, but floor space and storage space are limited. There is no administrative provision for taking care of one section of the class while the teacher works with the rest. The teacher has these 30-some different children in one classroom, and is faced with the challenge of providing arithmetic and justice for all.

What now?

The primary requisite is a state of mind—a state of mind which will accept as postulates (1) that everybody can think (2) that not everybody can think in the same way and (3) that no one method of thinking is, per se, better than another. These postulates imply the conclusion, then, that no one approach to arithmetic learnings and no one standard of performance should be applied to all pupils.

The implementation of this conclusion is simple indeed. You build for *all* pupils a few basic concepts, build them very firmly indeed: the meaning of a number (a whole number, a common fraction, a decimal fraction as the grade level progresses); a real understanding of the decimal base and the positional aspects of our system of numerical notation; the meanings of the fundamental processes and their interrelationships; a carefully-developed understanding of necessary vocabulary, not a parrot-like repetition of a textbook definition. Teach each new lesson in your usual way, including as many approaches to the situation at hand as may be suggested by the members of the group. Then, when the time comes for the practice sessions, let each child do it *his* way as long as his way is not incorrect and represents the most mature level at which he can perform comfortably and accurately. Note that this statement does not imply a wide-open permissiveness; it does imply that the teacher must not be incurably addicted to the "standard" method for doing things in arithmetic. After all, even the standard algorisms for any of our computations are standard, not because they are the best way of doing any one example, but because they apply to the general case in their particular category.

To illustrate how letting the child do it his way provides for individual differences automatically, in a manner of speaking. One day I saw a fifth grader doing division exercises with two-place divisors, three-place quotients with a fraction expressed in lowest terms, apparent quotients incorrect, and writing only the answers. He did twenty such examples with one mistake. I pointed it out to him saying, "I believe you have a mistake here." He looked at it for a couple of seconds and said, "So I do. This fraction should be $\frac{2}{5}$, not $\frac{2}{3}$. I am so used to writing $\frac{2}{3}$ that I wiggled my pencil the wrong way when I came to write the 5." In the same class, another child was writing out each trial for each quotient figure as "side work," changing each fraction to lowest terms by again recording each of his divisions, obviously understanding what he was doing, and getting the correct answers also. The teacher, bless her, accepted the work of each of these children as a matter of course, just as she accepted the intermediate levels of performance exhibited by other members of the class. Is that providing for individual differences? I think it is.

By way of contrast, there was a seventh grade class in which the teacher explained to me that nobody got away with any foolish-

ness in his class. He believed in teaching one way of doing a given kind of exercise and he wanted every step of this way recorded on the paper. The day's lesson was on the perimeter of rectangles. The formula that found favor was $P=2L+2W$. In finding the perimeter of a rectangle whose length is 18 inches and whose width is 12 inches, the pupils were required to write:

$$P=2L+2W$$
$$L=18'', W=12''$$
$$P=2\times 18''=2\times 12''$$
$$P=36''+24''$$
$$P=60 \text{ inches (ans.)}$$

Is there anything wrong with this procedure? Not at all. But one youngster drew a picture, labeled it, and wrote beneath it:

$$18''+12''=30''$$
$$2\times 30''=60''$$
$$P=60''$$

The teacher said, "Where are you getting this stuff?" The youngster said, "Perimeter means the distance around a figure, so I start at one corner and go around. When I get this far (the opposite corner) I see that the rest of the figure has the same measurements repeated, so I double the 30. It's easy that way. Actually the only thing I would have to write is the answer." The teacher explained very patiently: "In this class, we do not pull our answers out of the air. We start with the formula. We write the given value for each of the letters. We substitute in the formula. We work it out step by step. You do these examples over now, and do them as you should."

Now, I ask you, if someone really understands the meaning of *perimeter*, does he really need anything else besides the requisite dimensions to find the perimeter of any polygon? And whose idea was better from the point of view of transfer of concepts and skills to finding perimeters of other figures? I ask you another easy question: Why did almost every bright youngster in that class spend most of his time writing 100 times or more "I must behave in mathematics class?"

The foregoing are just two illustrations which could be multiplied many times. I do not advocate that the standard algorisms for performing various types of computations or that model solutions for certain types of problems should not be taught. I do advocate that if a child cannot deal adequately with the standard algorism, he should be allowed to use a less mature form, and that if another child can perform at a level more mature than what is commonly called "adult level of performance," he should do it his way. A great deal of research has been concerned with problem solving. Thiele[2] compared three different methods of teaching children to solve problems. The method in which the children were given the most leeway showed the best results of the three methods studied, and the conclusion is drawn that it is far better to give children plenty of experience in problem solving by their own devices than to impose upon them artificial procedures which they cannot understand or use effectively. Burch[3] investigated success with the formal steps of problem analysis in arithmetic as a contribution to solving problems. Among his conclusions is, "Attempting to follow the step-by-step procedure may actually lead to a lower level of performance on the total task." In general, the research on problem solving shows no significant superiority of any one method of approach over other methods, and attributes whatever superiority may appear to the skill and zeal of the teacher. So with problem solving, there is plenty of evidence in support of a multiple approach, of letting the child do it his way.

Give the Pupils a Chance to Explore

You probably read about a biology teacher in the Carmel (Calif.) High School, Miss Enid Larson, who enjoys phenomenal success with her students. One of her theses is "Just give a youngster an idea, and then get out of his way." I would add a corollary: If a youngster has an idea of his own, stay

out of his way. Encourage a spirit of inquiry, encourage independent thinking, encourage each child to have confidence in and respect for his own way of figuring things out, and an effective method of dealing with individual differences is in operation.

Another very effective technique for providing for individual differences is to have the pupils make up problems of their own. The stipulation common to all such assignments is that each pupil be able to solve the problem which he himself makes up. Then the problems may be passed around to others in the class, with the teacher's approval. The assignment may be to make up a problem using certain numbers, or to make up a problem about a given topic, or to make up a problem which can be solved by addition, and so on, or to make up just any problem. The results are fascinating. Every teacher ought to try this at least once, if for no other reason than that it is so much fun for the teacher. He learns things he never suspected about the children in his class, things about their interests, about the maturity of their thinking, about their ability to express themselves clearly. He gains first-hand evidence to confirm or to refute widely-held beliefs about how children think and about what interests them. He sees many opportunities for correlating the arithmetic program with the language arts, the social studies, or the science programs, but most of all, he can diagnose strengths and weaknesses of his pupils with regard to their functional competence in arithmetic and take appropriate steps to remedy, to provide further practice, or to challenge. Particularly in the last category, the pupils will provide much of their own material, given the opportunity to make up problems for themselves.

It is a great temptation to read a long list of pupil-made problems, but we must be satisfied here with a few samples. First, two problems made up at the same time in the same eighth grade class:

(a) Richard is eighteen years old and Larry is nine years younger than him. How old is Larry?

(b) Three brothers organized the Brown Furniture Co. The corporation was valued at $400,000. They sold 4,000 shares of stock at $100 each. The dividend on each share was 5%. After one year how much dividend will be paid to: John Brown on 1,200 shares? Sam Brown on 950 shares? Don Brown on 850 shares? Mary Smith on 500 shares? Mr. White on 300 shares? John Doe on 140 shares? Mrs. Jones on 60 shares?

Do you think that, with just this much evidence, you call tell something about the range of arithmetic abilities in that class?

Now, notice the presence or absence of realism in the following:

(a) Betty Cassidy bought a jacket originally priced at $15.95. She received a discount of 40%, because she worked in the store. How much did she save?

(Grade 8)

(b) Find the volume of a rectangular prism whose length is 3 kilometers, width is 4 kilometers, and height is 2 kilometers.

(Grade 8)

(c) A scout troop left Quincy at 9:00. Hiking at the rate of 3 miles an hour when will they get to Boston which is 10 miles from Quincy?

(Grade 6)

(d) How much will Bill save by buying $\frac{3}{4}$ pound of steak already chopped at 98¢ a pound instead of having it chopped to order at 105¢ a pound?

(Grade 6)

The great variety of problems which usually come from a class which is asked to make up any kind of problem they want to will often furnish a more comprehensive review of material already taught than the teacher could devise without a great deal of study. Frequently, also, some of the problems will involve concepts and skills not yet taught in the class. The fact that some of the children can make up and solve such problems may be a tribute to the teacher's success in building the basic concepts of which I spoke earlier.

Two-step problems had not been taught at the time this problem was written by a second grade boy:

I had 4 cents.
I got 6 cents from my mother.
I spent 3 cents.
How many cents do I have left?

He solved the problem correctly, first adding, then subtracting.

The fifth grader who made up the following problem had been taught nothing about multiplying fractions.

Each soldier has $53\frac{5}{17}$ pounds of equipment. How much weight will be on a truck which carries equipment for 12 soldiers?

This youngster had done some elaborate figuring before arriving at the weight of $53\frac{5}{17}$ pounds. He did most of that mentally, and I did not want to ask questions because he did not realize that I was watching him, but I did take down his solution for the problem. He wrote it this way:

$$\begin{array}{r} 53\frac{5}{17} \\ 12 \\ \hline 106\frac{10}{17} \\ 53\frac{5}{17} \\ \hline 639\frac{9}{17} \text{ lbs.} \end{array}$$

This is all he wrote on his paper. Study it for a moment. It is very revealing. And isn't this a more efficient and a more meaningful way to find the product of these particular numbers than the way in which pupils are usually taught to multiply them? What would you do for this boy?

Now, just one more example to show the value of pupil-made problems in diagnosis of a different sort. This one was made up by a sixth grader whose teacher might not otherwise have found out what seeing this problem told her. What would you do for this girl?

Mr. Jones is building a house. It is to be 10 cu. feet tall and 9 cu. ft. wide. How many cubic feet will that be in all? (10 cu′×9 cu′)

Each Pupil Is an Individual in Learning

In addition to the foregoing techniques for providing for individual differences among pupils, there are a number of activities the teacher may prepare which will furnish further practice, challenge, or opportunities for developing appreciation for the value of mathematics in our culture. These activities may be classified roughly as follows:

(1) *Puzzles, tricks, games, magic squares, and so on.* Often these activities may come under the general heading of painless drill. For example, once the youngsters know what a magic square is, you can leave some of the squares blank, and they will cheerfully do the required additions and subtractions to fill them in. Many games and puzzles are available from commercial sources. Others may be made by the teacher or the pupils. If you chose activities which incorporate self-checking features, groups of pupils can accomplish a great deal of productive practice with a minimum of teacher supervision. I have yet to see a pupil, regardless of his level of ability, who did not enjoy these puzzles, tricks, and games.

(2) *Correlations with other school subjects and with community life.* Most of these activities involve consulting reference books, or gathering data from other sources, and then making a report to the class. Such projects are valuable in developing appreciation for the usefulness of mathematics and for its contributions to our culture, but should not be used to the extent that they become a vehicle for keeping the bright students busy in the library and out of the classroom teacher's hair. It is the rare correlation activity that does anything to develop anybody's number insight.

(3) *Study of selected portions of the history of mathematics.* Most children are fascinated by tales of how the Egyptians wrote their numbers, of how measuring was done before the adoption of standard units, of "old-fashioned" ways of computing, and so on. Historical perspective is always valuable. For one thing, it may develop an appreciation for the convenience of our contemporary forms.

(4) *Activities which look ahead to something not yet taught.* There are many opportunities for extensions of concepts and skills which pupils can handle without help. For example, once the youngsters really understand multiplying by two-place multipliers, most of them can figure out how to multiply by three-place multipliers, or indeed, by multipliers of any size. This produces a feeling of real

satisfaction. It helps also to develop a propensity for self-reliance on the part of the pupils.

(5) *Embryonic number theory* (for want of a better term). We can devise specific examples of very interesting relationships which actually illustrate fundamental principles of mathematics. For example, the pupils may be guided to discover that in multiplying a number by 13, they do not have to use 10 and 3 for finding the partial products. They may use 9 and 4, or 8 and 5, or 2 and 4 and 7, or any combination of numbers which add up to 13. The general idea here has wide application. Or the pupils may be asked to take any magic square, double each of the elements, and see what happens. Do they have another magic square? How do the sums of any corresponding rows, columns, or diagonals in the first and second squares compare? Or have them list the multiples of 9, then add the digits in each of the numerals: 9, 18(1+8=9), 27(2+7=9), 36(3+6=9), and so on. In each of these activities, almost every pupil in the class will be able to see *that* it works. Some will be satisfied with that much. Others will wonder *why* it works and will be able to explain it. Still others will want to *experiment* with other similar ideas. In the case of the multiples of 9, for instance, they'll try multiples of 8, or multiples of 11, or even multiples of multiples of 9. In the case of the magic squares, they will try changing the elements by adding, subtracting, or dividing by the same number. You will find that it is not only the pupils with very high I.Q.'s who will do this experimenting. Everybody can think; remember? They will make all sorts of discoveries and will be thrilled and delighted. The teacher who sees the expressions on the faces of children who have made a discovery of their own will have one of those experiences which keep teachers in the profession. Furthermore, having developed the habit of inquiry, there's no telling what they will come up with on their own. Sometimes it is something which the teacher may not have noticed before, but that's all right—teachers can think, too.

If creative thinking in mathematics is to be nurtured, this last type of enrichment activity is perhaps the most effective nutrient. If I had my way, we would teach some of the axioms of general number and the Commutative, Associative, and Distributive Laws, by name, in the fifth grade. I would not *expect* any fifth grader to master these ideas, but some of them would, and these pupils would be able to produce explanations which would approach a formal proof for some of the relations which they discover. The others would at least become acquainted with the terminology and would absorb some of the general ideas, and then when they encountered the axioms and laws in the mathematics courses which surely will be a part of the high school curricula in the very near future, they would not be blocked by what seemed to them a foreign language.

If it is inevitable, as the scientists believe, that we are rapidly approaching the time when there will be no place for the unskilled worker, it would be the rankest sort of injustice to allow any of our young people to get through school without a firm faith in their ability to handle quantitative relationships. For quantity is everywhere. Most of the current research in the social studies is metric, in one way or another. Much of the research in biology is quantitative. In fact, a biologist of considerable stature told me that he would not be surprised if one of the key factors in conquering cancer turned out to be some relationship between the surface area of a cell and its volume. Linus Pauling thinks that mental illness will soon be proved to be chemical in origin and tied in with chemical abnormalities inherent in the genetic constitution of the individual. Even writing novels or poetry, or painting pictures which reflect the age in which one lives implies an understanding of the dominant forces of the times.

We may do our part by beginning at once, in the situation in which we find ourselves now, to provide a nourishing and digestible fare of arithmetic learnings for all pupils. We must do our best to dispel the notion

that only a near genius can be competent in arithmetic. We must do our best, also, to impress upon all our students that they not only *can* learn arithmetic; they *must* learn it, in justice to themselves.

It is not necessary to provide multiple-track curricula. In an article called "Mathematics for the Future" which appeared in the March issue of *The Kansas Teacher*, Howard Fehr[4] states his hypothesis that elementary portions of mathematics for the future will be the same for all pupils. He adds, "It is merely longer time, with more concrete illustrations that are needed for the slow learner, and not a different type of curriculum."

Summary

We have discussed three techniques for providing for individual differences in one classroom: (1) Let a child use his own ideas, within limits. (2) Have pupils make up problems for themselves and other pupils to solve. (3) Provide special enrichment activities of a variety of types. None of the things we have mentioned require unreasonable amounts of the teacher's time. On the contrary, they encourage the children to be independent and self-reliant in the matter of dealing with numbers, using their own brains for their own good.

Bronowski's article which I cited earlier ends as follows: "We can steer our society only if we combine a general knowledge of where we are bound to go with a specific will of where we want to go.

"This is the true art of the statesman in the age of atomic energy, biological control, and automation. And it is relevant to recall that another word for the control of automatic machines today is the word 'cybernetics,' and that this is a Greek word for the art of steering a ship."

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Editor's Note. Mrs. Phillips asks for the use of imagination coupled with understanding in the arithmetic classroom. She points out that each child can think but that not all will think alike in pattern or in depth and that it is our job to stimulate and encourage thinking. She says that each child not only can learn arithmetic but must learn it. She gives us a challenge to foster this type of learning in the regular classroom. Again, it is the role of the teacher that is so very important in setting the stage for a learning that uses thinking and discovery and not only permits but encourages pupils to explore ideas. This is very different from the older pattern in which the teachers states "You must do it this way or you will get it wrong." But this newer approach requires thinking and understanding on the part of the teacher. Is this wrong? With Mrs. Phillips, let us accept the challenge of educating our young people for a proper role in our society of 1960 to 2000 instead of 1900 to 1950.

New York's Program for Teachers

The State Education Department of New York approved some twenty special programs in science and mathematics where elementary school teachers could study backgrounds of mathematics (not methods) this past summer with tuition and living expenses paid by the state. During the next school year, the program will be continued at a larger number of centers spread throughout the state. Of course, this program cannot reach all teachers but the total number will be in the thousands. What are other states doing to enlarge the mathematical backgrounds of their elementary school teachers?

Low Visibility, or, Ideas in a Fog*

R. L. Morton
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We all know that some children find arithmetic difficult. It is commonly assumed that children find arithmetic difficult because of an inadequate background of experience for the topic presented, or because they are relatively immature, or because they have insufficient native ability. Each of these reasons is doubtless a valid one in many cases.

However, those who fail to make satisfactory progress are not always those with inadequate experience, or insufficient maturity, or not enough ability. I am convinced that many of the troubles of young learners in arithmetic are due to the language in which arithmetical ideas are expressed, the language of teachers and textbook writers. Ideas are too often "invisible" because they are concealed in a fog of words.

The activities in which I engage require considerable travel, most of it by air. I have become interested, in an amateurish way, in the techniques for handling airplanes in bad weather. From my seat in the rear of the cabin I can see that the wing flaps have been let down and the landing gear has been lowered but the overcast (fog) is so thick that at times I can not see the tips of the wings. I know that we have left the holding pattern and are coming in for an instrument landing. I know that the pilot can't see any farther than I can but I also know that he has an advantage which I do not possess. Two beams, coming up from the runway at a 3-degree angle, register on a dial in front of him. One beam, which controls a vertical needle, indicates whether the airplane is headed for the center of the runway or is too far to the left or too far to the right. The other beam, which controls a horizontal needle, indicates whether the airplane is on the glide path or is above it or below it. A purple light flashes; we are passing over the outer marker, $4\frac{1}{2}$ miles from the end of the runway. A minute or so later an amber light flashes; we are passing over the inner marker, 3500 feet from the end of the runway. We are now 300 feet above the ground. If there is a 300-foot ceiling, we break through the overcast at this point and are assured by the two rows of lights which bound the runway just ahead. We have made a successful landing, a routine landing, in spite of the fog.

But such landings are for experts. The situation just described is not one in which a beginner should assume full responsibility. Beginners are not taught the rudiments of flying under conditions of low visibility. The analogy should be clear. We who classify ourselves as experts have no difficulty with the ideas of arithmetic even though they are shrouded in a fog of words, but for beginners, there should be no fog; visibility should be unlimited.

Some years ago, I acquired a copy of Robert Gunning's book, *The Technique of Clear Writing* (McGraw-Hill). This book was not prepared for teachers. It was written for business men, advertisers, public relations men, reporters, editorial writers, lawyers and judges, social workers, economists, and other like groups. Gunning says that our "Heads buzz and eyes burn" as we try to read "Tax forms, reports of Congressional committees, memos from the legal department, instructions on how to assemble a boy's workbench." Gunning has much to say about the fog caused by long sentences and big words, words of three or more syllables. He develops a measure of readi-

* Paper read at the convention of the *National Council of Teachers of Mathematics*, Cleveland, April 11, 1958.

bility which he calls the "Fog Index." I am tempted to quote much from this interesting book but I shall content myself with three short quotations. These three quotations are paralleled with translations into easier reading, the translations by Gunning.

"Hard Reading for College Graduates*"	"Easy Reading for Most People*"
Consumer elements are continuing to stress the fundamental necessity of a stabilization of the price structure at a lower level than at present.	Consumers keep saying prices must go down and stay down.
The finance director related that substantial economies are being effected in his division through increasing the time interval between distributions of data-eliciting forms to employing business entities.	The finance director said his division is saving money by sending fewer questionnaires to employers.
The establishment of a uniform level of freight rates on wool no higher in relation to distances to Southern consuming points than those to Eastern and New England consuming points is necessary in order to avoid imposition against Southern consumers of an added disadvantage of higher rates over and above the normal disadvantage attributable to their greater distances from source of this raw material."	If freight rates on wool are higher in the South than in the East and New England, the Southern consumer will have a double handicap. For, even with the same rates, the Southerner pays a larger hauling charge because he is farther from the source of supply."

Before turning to the language used in arithmetic instruction in the elementary school, let us consider briefly the language of materials designed for teachers. I have been amazed at times, when reading such materials, at their needless difficulty. College professors who write college textbooks sometimes seem to strive to make their books hard to read. I find myself wondering, as I read these books, if the authors are not trying to impress their colleagues rather than educate the students. They seem to be striving to impress others with their wisdom rather than to make their readers wise.

Recently, I read some of the manuscript for the forthcoming yearbook on arithmetic, a yearbook of the National Council of Teachers of Mathematics. One of the chapters which I read is, in my opinion, unfit for inclusion in this yearbook and I have said so in rather emphatic language. A major objection to this chapter is the language used, although there are also other objections. I do not yet know what decision has been made with reference to the use of this manuscript.

* From "The Technique of Clear Writing," (McGraw-Hill) copyrighted by Robert Gunning.

I have collected a number of interesting examples of difficult language which appears in materials prepared for teachers but we do not have time to consider them. Let us turn now to language used in elementary-school classrooms, language designed for the eyes and ears of children.

A third-grade pupil read this about the number 222. "The place in which each 2 is written gives each 2 its name and its meaning." Very few eight-year-olds can understand that sentence, assuming that they can read it.

Too often, the statement of a problem ends with a question which is asked in formal stilted, and therefore difficult, language. Here are samples, picked up in Grade 3.

"How many more dolls than Ruth has Jane?" Wouldn't it be much better to say, "Jane has how many more dolls than Ruth?"

"How many rabbits in all had he then?" Why not say, "How many rabbits did he have then?"

"How many pennies did Mary then have in all?" Why do so many teachers use such stilted expressions as "then have in all?" If "in all" is deleted, say "have then" instead of "then have." The question becomes,

"How many pennies did Mary have then," or, "Then how many pennies did Mary have?" Let us cease asking such questions as, "How many pennies did Mary then have in all?", especially in Grade 3.

In another third-grade classroom this occurred: "Deal out 18 blocks into three equal groups." Most of the pupils did not understand at all.

A fourth-grade class was trying to learn to divide by two-place numbers. This is difficult, too difficult for most fourth-grade pupils. It was made more difficult than it needed to be, however, by the example which was selected as a model and the accompanying development. The pupils were to divide 676 by 13. $13\overline{)676}$ They were instructed as follows: "Think: 6×10 (in 13) $=60$, and subtracting leaves 7 of 67. Is 7 large enough to contain 6×3? No; so 6 is too large to use. Try 5." This example epitomizes difficult language and also a very poor teaching procedure.

In an effort to teach fifth-grade pupils certain principles used in writing Roman numerals, two rules are stated as follows:

"If a letter is repeated, or if a letter is followed by a letter of less value, the number is the sum of the values of the letters.

"If there are two different letters and the one with the smaller value is written first, the number is the difference of the values of the letters."

This is not bad for intelligent adults but it is much too difficult for fifth-grade children.

Here is a sample from Grade 6. A brief development on finding the area of a rectangle terminates with this italicized statement. "*To find the area of a rectangle multiply the length by the width. The length and width must be given in the same unit of measure. The answer is given in square units of that measure.*"

The first sentence is not bad. But the best thing to do with the other two sentences at this stage is to delete them. Save that for next year and then say it more simply. Why complicate the situation and provide difficult reading by talking about same and different units of measure and square units of measure when the topic has just been introduced?

I could quote many more examples of difficult language which have been picked up in schoolrooms. I probably have 100 or more in my collection. But in the time which remains to me I wish to present some samples of language of a somewhat different kind. I refer to language which is not so difficult in itself but which represents a misuse of words. When words are misused, wrong ideas are likely to develop. Hence, a needless difficulty in learning is introduced. There is a surprising amount of this sort of thing going on in schoolrooms, far more than there should be.

A fourth-grade teacher said: "Let us see how many groups of 2's we can get from 13." This statement implies that each group that we get from 13 will contain a certain number of 2's; it is not clear how many 2's there will be in each group. What is meant, apparently, is, "Let us see how many groups of 2 each we can get from 13." Or, we might say, "Let us see how many 2's we can get from 13." The statement, "how many groups of 2's we can get from 13" is incorrect and is very likely to be misleading.

A little later, the same teacher wrote on the chalkboard, "Find $2\overline{)13}$." What is the pupil to find? The teacher told him to "find $2\overline{)13}$." Apparently, she did not want him to find $2\overline{)13}$ at all but to find how many 2's there are in 13, that is, to find *the quotient* if 13 is divided by 2. All a pupil should have to do when told to "find $2\overline{)13}$" is to look at $2\overline{)13}$ and say, "There it is; I have found it."

How should a pupil read, "Find $2\overline{)13}$"? Try to read this yourself. I hope that you do not think, "Find 2 into 13," an expression which is completely meaningless.

A fifth-grade class was being introduced to the subject of averages. They had a problem that stated that the number of pupils absent on the 5 days of a certain week was 3, 1, 1, 3, and 2, a total of 10. The teacher said: "To find the average number absent

each day, we divide the total number, 10, by the number of days, 5." Of course, there is no such thing as "average number absent each day" unless the number absent each day is the same. In that case, there would be no need for computing an average. The number absent each day would be the average. Time and again, I hear this expression "average each," an expression which is incorrect and misleading. What is meant, of course, is, "average per day." Later, in dealing with a problem about rate of travel, the same teacher said, "How many miles an hour did he average?" This is good. She did not say, "How many miles each hour did he average?" But still later, in solving a problem about collections made for a charitable cause, this teacher said, "Find the average amount collected each day." This is bad.

Some faulty language expressions probably do not mislead pupils. For example, a sixth-grade teacher said: "To what common denominator can we change $\frac{2}{3}$ and $\frac{3}{4}$?" Most of the pupils probably knew what the teacher meant although, of course, it is not possible to change fractions to a common denominator. We change fractions to like fractions, or to common denominator form. Or, we might say that we change fractions so that they have a common denominator. This slip occurs very often in the language of teachers and writers. It is easily avoided and it would seem that we should avoid it.

Another teacher said: "What fractional part of the carton of eggs is filled?" She should have said: "What fractional part of the egg carton is filled?" Of course, "carton of eggs" means that the carton is full of eggs. However, it is doubtful if the pupils were misled.

A popular redundancy is illustrated by the following: "Multiply 17×25." Inasmuch as $17 \times 25 = 425$, the expression, "Multiply 17×25" really means, "Multiply 425." The correct expression is, "Multiply 25 by 17," or, "Find the product of 17 and 25."

About 35 years ago, the late P. R. Stevenson, then a member of the staff of the Bureau of Educational Research at Ohio State University, told of asking a pupil what he was then having in arithmetic and receiving the reply, "We are having *guzzintos*." He said it as if it were spelled g-u-z-z-i-n-t-o-s. "You are having what?" asked Stevenson. "Guzzintos," the boy replied. "You know, 5 goes into 20 four times." People smiled as Stevenson told his story. However, it must be admitted that at that time textbooks as well as teachers used this "goes into" lingo.

A few minutes ago, I expressed the hope that you are never guilty of reading "Find $2\overline{)13}$" as "Find 2 into 13." Such expressions as "2 goes into 13," "divide 2 into 13," "2 into 13," and, perhaps the most terrible of all, "take 2 into 13," are meaningless. They should not be used. Fortunately, these ex-expressions are disappearing from the literature. Recently published textbooks are almost completely free of them. But they do persist in the language of a good many elementary teachers and a few writers. A book for elementary teachers, published in 1957, contains more than 90 instances of such language. Fortunately, the book has other attributes which, in my opinion, will prevent its wide use. Even the National Council of Teachers of Mathematics, through its official publications, seems to countenance the continued use of such expressions. Just a few years ago, an article in *The Mathematics Teacher* contained such expressions and in the February 1958 issue of THE ARITHMETIC TEACHER I find "taking 8 into the 8 of 800," "8 will not go into seven," and "8 is divided into 79 tens."

In the March issue of THE ARITHMETIC TEACHER, even more flagrant instances occur. Here are some direct quotations:

"Twelve into 24 goes two times."

"12 into 7 goes no times."

"12 goes into 75 six times plus."

On the two facing pages on which the article under consideration appears, there are in all 13 instances of this "goes into" lingo. In no one of the 13 cases would it have been difficult to express the essential idea clearly and correctly.

I hasten to add that I do not wish to be critical of the work of the editor of THE ARITHMETIC TEACHER, Dr. Sueltz. I believe, however, that it would be well for The National Council of Teachers of Mathematics, through its official publications, to set a good example in the use of quantitative language. I see no good reason for the continued use of faulty division language in our publications. And, I might add, I see no good reason for saying "percentage" when we mean "per cent," a slip which occurs in the March issue of THE ARITHMETIC TEACHER, or for saying "proportion" when we mean part, or fraction, or portion, a mistake which occurs frequently in educational publications, although I have not seen it recently in either of our journals.

Now, I don't wont to be so elementary as to bore you but I must remind you that there are only two fundamental ideas represented in most division situations—the measurement idea and the partition idea. The measurement idea is exemplified by this situation.

Mother has a dozen eggs. She is baking cakes for the church bazaar. Each cake takes 3 eggs. How many cakes can she bake? How many 3's are there in 12?

In this situation, the child is to divide 12 eggs into groups of 3 each. He is to discover how many groups there will be. He is to divide 12 into 3's. By no stretch of the imagination is he to divide 3 into 12, or take 3 into 12, or find 3 into 12, or find how many times 3 goes into 12.

The partition idea may be illustrated by this situation.

There are 12 pieces of candy in a box. Three children are to share this candy equally. How many pieces are there for each child? What is one-third of 12?

In this situation, the child is to divide 12 pieces of candy into 3 equal groups and is to discover how many pieces there will be in each group. He is to divide 12 into 3 equal groups or parts. By no stretch of the imagination is he to divide 3 into 12, or take 3 into 12, or find 3 into 12, or find how many times 3 goes into 12.

It is high time we teachers and writers discontinued the use of such expressions. They are bound to cause temporary confusion when the ideas of division are first presented. I continue to be amazed at how well children learn in spite of the hurdles which we place in their paths. At the risk of tramping on your sensitive toes I must add that these and other unfortunate expressions persist not because teachers believe they facilitate learning but because the same teachers can not or will not break their own bad habits. I know from experience how difficult it is to discontinue the use of a pet expression which we have used for years. I committed these division sins for many years.

As a clincher argument for any one who is reluctant to give up such an expression as, "Divide 3 into 12," what parallel expression would he use if he wanted to have two pies cut so that each of the pieces is one-fifth of a pie? Would he say, "Divide $\frac{1}{5}$ into 2 pies?" That is similar to "Divide 3 into 12." Of course, he would say, "Divide (or cut) 2 pies into fifths."

The word "dividend" means "a number to be divided." When 12 eggs were divided into groups of 3 each, obviously 12 was the dividend. But those who say, "Divide 3 into 12" make 3 the object of the verb "divide." In other words, "Divide 3 into 12" not only represents an impossible physical situation and is mathematically meaningless but it is also bad grammar.

The challenge of arithmetic should lie in the arithmetic itself and not in the language in which arithmetical ideas are expressed.

As a final fling at this matter of language and learning I should like to offer the observation that one reason some children do not learn more is that some teachers use too many words. Some teachers talk too much. A chatter-box type of teacher once said to me: "I explain and explain and explain and still they don't get it. They surely are dumb." Those children were not dumb; they were numb.

EDITOR'S NOTE. Professor Morton's plea for good language usage in arithmetic is well taken. The editor readily admits that he has permitted to be printed certain types of semi-technical expressions that he would not use. It is heartening to note that colloquial and crude expressions are disappearing from our texts and reference books. It is so easy to write according to a pattern of speech instead of consulting a standard dictionary. The editor was a little amused at Professor Morton referring to a "lingo" and to write "Each cake takes 3 eggs." However, we would all be aiding the teaching of arithmetic if we would be more careful of the language we use. We should be precise, we should say exactly what we mean, and we should use language that is at least reasonably acceptable in grammar and syntax. But who is to be the arbiter in the use of technical expressions that have as yet not acquired a standard accepted form? Do all agree with Professor Morton's criticism and his preferred usage? Whether we agree or not, his main thesis is one with which we should agree.

Christmas Meeting in New York City December 29–30, 1958

The Eighteenth Christmas Meeting of the National Council of Teachers of Mathematics will be held in New York City on December 29 and 30, 1958, at the Sheraton McAlpin Hotel (Broadway at 34th Street).

Elementary school talks will deal with basic issues in the teaching of arithmetic, issues in teaching the gifted, ways to provide for individual differences, preparation of elementary school teachers, new curriculum and subject matter emphases, ways to teach mental arithmetic; etc.

Junior High School talks will deal with basic issues, eighth grade algebra, providing for individual differences, providing for the able, accelerated mathematics programs, the University of Maryland Project.

Other talks will deal with the secondary school curriculum, college freshman mathematics, recommended changes in college mathematics, providing for individual differences at the high school level, selection of students for advanced mathematics programs, actions of government and professional groups affecting mathematics teaching (A.A.A.S., N.S.F., N.C.T.M.), and calculus, algebra, probability and statistics, and geometry in the secondary school.

General session speakers will include Dr. Conant, Dr. Newsom, and Frank Allen.

There will be T.V. programs and film presentations.

Recipe for Arithmetic

1 cup intelligence
2 cups thinking
$\frac{1}{2}$ cup addition
$\frac{1}{2}$ cup subtraction
$\frac{1}{2}$ cup multiplication
$\frac{1}{2}$ cup division
Dash of mistakes
$\frac{1}{4}$ cup corrections
1 heaping cup effort
$\frac{3}{4}$ cup decimals
$\frac{3}{4}$ cup fractions
4 cups problems
4 cups answers
$\frac{1}{4}$ cup graphs
$\frac{3}{4}$ cup measures
1 cup per cents
1 heaping cup practice
1 tsp. digits

Sift the intelligence and thinking together. Add the addition, subtraction, multiplication, and division; mix well. Fold in fractions, decimals, and per cents and insert mistakes sparingly. Mistakes may be left out without ruining the texture. Use effort. Fold in slowly the measures and graphs. Add the digits. Work in the problems and effort until well mixed. Insert answers and corrections. Let simmer until meaning is clear. An extra portion of practice may add to the seasoning. Makes as many servings as needed. Guaranteed to give excellent results.

Contributed by MARA ZEGELIS (Grade 7)
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Teaching the Number System Inductively

J. Allen Hickerson
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I WAS ASKED A QUESTION at the conclusion of my address at the Cleveland Conference of the National Council of Teachers of Mathematics last April. The question deserved a fuller answer than I was able to give at the time. I shall attempt to answer it here hoping that the questioner and other members of the audience may have opportunity to read this reply.

Although the topic of my paper was "Teaching Arithmetic Symbols as Representations of Concrete Quantitative Situations," the question had to do with teaching the number system. The question was—"The following statement appears in your book:* 'A systematic study of arithmetic as a number system should be made . . . in the secondary school and college, not in the elementary school.' Why do you believe this?"

There are several reasons why I believe pre-adolescent children should not engage in a systematic study of the number system. Before giving my reasons, however, perhaps it should be pointed out that a systematic study of the number system in my mind can be compared with a systematic study of grammar.

Children *first* learn to use language correctly from hearing it and speaking it correctly—not from studying grammatical rules. Only after years of usage in everyday experiences is grammar as grammar taught systematically.

First of all, pre-adolescent children think mostly in terms of people and things—in fact, people and things that are immediately about them. For the most part they are not interested in nor can they understand abstractions and symbolism. Symbols, to *mean* anything to most children, must represent something they can see, touch, smell, taste, hear. A symbol associated frequently enough with an object will in time come to stand for the object—just as the sound of the bell came to represent food to Pavlov's dog.

Even those arithmetic authorities who emphasize the importance of systematically teaching the number system to children in the primary grades, recognize that children think in terms of things. In teaching our system of numeration—ones, tens, hundreds, thousands, etc.—they resort to the use of such material objects as strips of paper in "pockets"; bundles of sticks; the abacus; structured blocks, notched sticks, frames, or pennies and dimes. In other words, they use concrete materials to represent numerical symbols.

May not this become a little confusing to the child? If, by definition, a symbol is a representation of something else (a thing, a relationship, an idea) why reverse this relationship and have children learn to look at things as representations of symbols?

One might argue, however, that a number symbol such as 23 may represent a collection of objects. On the other hand, 3 strips of paper in the one's pocket and 2 strips of paper in the ten's pocket can very well represent the numerals 3 and 2 in the number 23. Granted (so goes this argument) that a *number* can represent things. Things, however, can represent the separate *numerals* composing the number.

Nevertheless, things are things to children and not representations of something else—especially are they not representations of abstractions. By not having the relation-

* *Guiding Children's Arithmetic Experiences*. Prentice-Hall, Inc., Englewood Cliffs, N. J. 1952. p. 37.

ships between arithmetic symbols and things simple and clear from the start, the real nature of arithmetical symbols may become obscured from many children. It is extremely difficult, sometimes impossible, to clear up in people's minds misconceptions of arithmetic and mathematics which were acquired in the elementary school.

The mere fact that teachers resort to objects to try to help children understand the intricacies of our number system suggests that perhaps children are not intellectually ready to comprehend these intricacies.

Another reason why I believe that systematic teaching of the number system should be delayed until most people have reached adolescence is that such emphasis in the early years tends to focus the child's attention on the nature of the symbolism itself, rather than on that which the symbolism represents.

As far as most children are concerned their interests and capacities in arithmetic reside in (1) the quantitative aspects of the things and events in their everyday experiences and (2) in the manipulation of numbers in computation. Formal concentration on the third aspect of arithmetic—the nature of the number system istelf—is of concern to a comparatively few.

Comparing arithmetic with reading, over-emphasis on the nature of number symbols is similar to over-emphasis on the nature of word symbols. By concentrating on the words themselves a child tends merely to pronounce syllable after syllable and word after word in a passage. He does not get *meaning* from the printed page because he does not think of the things, relationships, or ideas which are being represented by the syllables and words which he is pronouncing. Concentration on the nature of word symbols and how they are pronounced tends to interfere with the meaning of the symbols, that is, what they represent.

Similarly, a child can learn to become so conscious of number symbols themselves that when he adds five 4-digit numbers, for example, he does not think of the things, relationships, or ideas which are being represented by the numbers. He just adds numerals column by column not caring or wondering whether or not his answer makes sense.

Try this with an adolescent or adult friend. Show him or her an addition containing five 4-digit numbers. Or show him such an arithmetical expression as $48\overline{)1432}$ or $8\frac{1}{2}\times1\frac{3}{4}=$. See what his first reaction is. If your experience is like mine, your friend will start in adding columns beginning with the one's. For the division or the multiplication of fractions he will try to remember how to get the answers and then proceed with the computation, using whatever techniques he was taught.

This immediate reaction of wanting to get an answer is most likely the result of the over-emphasis placed in schools on the numbers-as-numbers aspect of arithmetic and the under-emphasis on the *menaing* of numbers and the *meaning* of arithmetical expressions, that is, on what these expressions represent.

Children are capable, however, of learning to respond *initially* to such an expression as $48\overline{)1432}$ somewhat as follows: 1432 things are divided into 48 equal parts, or, the number of 48's in 1432 is about the same as the number of 50's in 1500, which is the same as the number of 5's in 150, or 30.

Children are capable of learning to react *initially* to $8\frac{1}{2}\times1\frac{3}{4}=$ like this: $1\frac{3}{4}$ taken $8\frac{1}{2}$ times is about 2 taken about 8 times, or 16.

The systematic teaching of the nature of the number system in the elementary school has another disadvantage. Habitually, viewing numbers as consisting of ones, tens, hundreds, thousands, etc., forces the young child to look at numbers hind-part foremost. To analyze numbers from right to left or to attack numbers from right to left while computing tends to over-emphasize the importance of the least significant digits of a number and under-emphasize the most significant digits. It thus tends to detract the child from an awareness of the real meaning of numbers which the left-to-right reading facilitates.

Moreover, right-to-left analysis of numbers and right-to-left computation necessitate an eye-movement direction just the opposite of the direction which young children at the same time are learning while building reading skills.

It is not until division is reached that children begin to view numbers frontwards, that is, from left to right. This "novel" way of looking at numbers may be one of the contributing causes to difficulties many children experience with division.

A great disadvantage of systematically teaching the number system in the elementary school is that such teaching of necessity is primarily *deductive* teaching. Little opportunity is afforded children to discover and to formulate their own rules and principles concerning the number system and computational techniques.

From the beginning children are *shown* or *told* that 10 is the basis of our number system. For example, instead of allowing children to *discover* through inductive teaching that 15 objects can be arranged in 2 groups in different ways (e.g., 6 and 9, 7 and 8, 8 and 7, 9 and 6, 10 and 5, etc.) it is advocated that children be *told* that

$$\begin{array}{rcl} 8 & & 10 \\ \underline{+7} & \text{is really} & \underline{+\ 5,} \\ 9 & & 10 \\ \underline{+8} & \text{is really} & \underline{+\ 7,} \end{array} \quad \text{etc.}$$

Instead of allowing children to make the natural inference that 23 is made up of 20 and 3, 78 is composed of 70 and 8, and so on, the advocates of number-system teaching wish children to be *shown* or *told* right away that 23 is composed of 3 ones and 2 tens, 78 is composed of 8 ones and 7 tens. In order for this to be done, however, the children must first be taught the 1 and 10 tables: $2\times1=2$, $3\times1=3$, $4\times1=4$ up to $9\times1=9$, and $2\times10=20$, $3\times10=30$, $4\times10=40$ up to $9\times10=90$.

These multiplications would usually not be learned in relation to the children's own everyday problem situations. They would be taught primarily in relation to the scrutinization of 2-digit numbers, since 23 cannot be analyzed into 3 ones and 2 tens unless children first know that 3 ones are 3 and 2 tens are 20. 78 cannot be analyzed unless children know that 8 ones are 8 and $7\times10=70$.

Discover and Experiment

Instead of allowing children to *discover* (and memorize) 20 and 10 are 30, 40 and 50 are 90, 70 and 80 are 150, etc., just as they should have discovered (and memorized) 2 and 1 are 3, 4 and 5 are 9, 7 and 8 are 15, etc.;

instead of allowing children to *discover* that such numbers as 42 and 56 can be separated into 40 and 2 and 50 and 6 and then added by combining 40 and 50, then 2 and 6, and then 90 and 8;

instead of allowing children to *discover* that there is more than one way of getting an answer and that right-to-left column addition is a computational trick which evolves from the logically and psychologically developed left to right method:

(a) L to R	(b) R to L	(c) R to L
42	42	42
+56	+56	+56
90	8	98
8	90	
98	98	

(a) L to R	(b) R to L	(c) R to L
		1
38	38	38
+59	+59	59
80	17	97
17	80	(Short cut)
97	97	

instead of helping children to *discover* that the reason why there are several ways

of obtaining an answer is because there are several ways of separating numbers into parts, computing with the parts, and then combining the partial answers;

and instead of encouraging children to *experiment* with various ways of separating numbers into parts, manipulating the parts, and putting the partial answers together,

							1
	38	38	38	38	38	38	38
	+59	+59	+59	+59	+59	+59	+59
for example,	98	99	88	89	80	17	97
	− 1	− 2	9	8	17	80	
	97	97	97	97	97	97	:

the number-system advocates usually start right in with the right-to-left technique by adding ones to ones and tens to tens. When "carrying" is necessary it is not called "carrying," it is called "regrouping." The term "regrouping" comes from the regrouping of strips of paper in ones-tens-hundreds pockets or sticks in bundles or beads on wires, etc. Thus, the children are *shown, told, taught one* way of getting an answer.

By insisting on the ones-tens-hundreds approach to computation teachers often discourage children from experimenting with numbers. They lead children to believe that there is one way of computing for an answer which is the *best* way for everybody.

The subtraction of 2-digit numbers offers a very good illustration of the "best method" emphasis of the number-system advocates. They *explain* and *show* to the child (deductive teaching) that in a subtraction such as

$$\begin{array}{r} 82 \\ -58, \\ \hline \end{array}$$

since we cannot take 8 from 2, we take 1 strip from the ten's pocket and place 10 strips in the one's pocket. Now we can take 8 strips from the one's pocket leaving 4 strips, and 5 strips from the ten's pocket leaving 2 strips.

The regrouping of strips is written in this way:

$$\begin{array}{r} 712 \\ \not{8}2 \\ -58 \\ \hline 24. \end{array}$$

Thus children learn *the* way to subtract. (When these same children become juniors or seniors in a teachers college or school of education they usually are very much surprised to learn that there are other ways of subtacting.)

Children who start subtraction with the left-to-right method can arrive at a right-to-left trick by developing the following sequence of computations. Each type of computation in this sequence should represent a problem-situation in the children's experience. Each type is also related to a type of computation previously learned.

(*Continued on next page*)

(1) (The basic combinations)

8	10	20	
−2	− 3	−10	etc.
6	7	10	

(2) (Multiples of 10)

80	90	40	
−20	−50	−30	etc.
60	40	10	

(3)

L to R	R to L	R to L
98	98	98
−52	−52	−52
40	6	46
6	40	(Short cut)
46	46	

(98 is separated into 90 and 8, 52 into 50 and 2.)

(4)

2
−6
−4

(Could represent a drop in temperature.)

(5)

10	40	80
− 4	− 4	− 4
6	36	76

(6)

L to R	L to R
92	92
−56	−56
40	44
− 4	
36	36

(92 = 90 and 2, 56 = 50 and 6)

(7)

Experimenting with other ways of separating 92 and 56:

(a)
92 = 82 and 10
−56 = −50 − 6
32 and 4 = 36

(b)
92 = 86 and 6
−56 = −56 −0
30 and 6 = 36

(c)
92 = 80 and 12
−56 = −50 and − 6
30 and 6 = 36

(d)
92
−56

92+10 = 90 and 12
56+10 = −60 − 6
30 and 6 = 36

Method (c) (decomposition) and method (d) (equal additions) can lead to the discovery of right-to-left techniques.

Another approach to a right-to-left technique which is built on children's previous learnings is based on the "additive" or Austrian concept of subtraction.

For

$$\begin{array}{r} 7 \\ -4 \\ \hline \end{array}$$

think 4 and what are 7? 3. This can be written

$$\begin{array}{r} 4 \\ + \\ \hline 7. \end{array}$$

For

$$\begin{array}{r} 74 \\ -32 \\ \hline \end{array}$$

think 32 and what are 74? This can be written:

$$\begin{array}{r} 32 \\ + \\ \hline 74; \end{array}$$

2 and what are 4? 2; 3 and what are 7? 4; the answer is 42.

$$\begin{array}{r} 92 \\ -56 \\ \hline \end{array}$$

can be thought of as

$$\begin{array}{r} 56 \\ + \\ \hline 92. \end{array}$$

Think, 6 and what are 12? 6; 6 and what are 9? 3. This is the so-called "additive-carry" method.

An advantage of starting 2-digit subtraction with the left-to-right approach is that it avoids teaching something which later must be unlearned. The right-to-left ones-tens-hundreds approach necessitates *telling* the child that in such a subtraction as

$$\begin{array}{r} 92 \\ -56 \\ \hline \end{array},$$

"We cannot take 6 from 2." Children know or can learn that when the temperature is 2° and drops 6° the new temperature is 4° below zero or −4°. They also know that in some games one can have 2 plus points and 6 penalty points given a score of minus 4. Why, then, tell children that we cannot take a big number from a little one?

$$\begin{array}{r} 2 \\ -6 \\ \hline -4. \end{array}$$

More Than One Way

Some teachers believe that children would be terribly confused if they were taught that there is more than one way of getting an answer. As it is, these teachers hold, many children can hardly learn even *one* technique of adding, subtracting, multiplying, or dividing.

This, too often, is true. It may be due, however, to certain remediable causes. Perhaps, first of all, the attitude of teachers from the kindergarten up can either encourage or discourage exploration and experimentation with numbers.

Over-emphasis upon accuracy and perfection in computation causes many children (and teachers) to hold on to *one* sure method, even if it is counting on fingers. Rewarding and praising accuracy and speed tend to stifle venturesomeness with numbers.

Children must have adequate intelligence as well as an experimental and inventive attitude to be capable of engaging in creative and original thinking with numbers. Some children would be mentally capable of experimenting with subtraction of 2-digit numbers, for example, if they only could wait another year before being taught the process.

Performing a computational process that is shown or demonstrated by the teacher or book and then drilled upon requires less mental ability than discovering or thinking it out for oneself and then drilling upon it. Since computational techniques *can be acquired without understanding* and *can be performed mechanically*, some teachers are tempted to teach a process before children are ready to understand it.

To be sure, some people may never be able to understand many of the intricacies of our number system. They require patient showing, telling, and demonstrating. *All* children, however, should not thus be taught deductively.

Those children who are potential mathematical thinkers should not be hindered from learning what creative mathematical thinking is. They should not be deprived of the opportunity to experience the joys of arithmetical discovery.

It is my opinion that basing computational understanding upon early systematic and deductive teaching of the place-value structure of our number system and that teaching children that there is a best way for all to add, subtract, multiply, or divide tend to stifle creative thinking and to instill in children the attitude that arithmetic is

difficult to understand. (This attitude is fostered when the teacher or the book is always telling or showing or explaining —and correcting.)

It is also my opinion that mathematicians should learn more about how children think; that elementary school teachers should learn more about arithmetic and its possibilities in enriching the creative lives of their children; that teachers in teacher-preparation institutions should learn how to help future teachers lose their arithmetical insecurities, dislikes, and fears; and that parents acquire a more positive attitude toward the mathematical potentials of their children and the teaching potentials of teachers.

The aim of all of us, I submit, should be the production of more and more people who are not afraid of mathematics, who have some understanding of and ability to use mathematics, and who think imaginatively and creatively in mathematics. Such people should be the products of our schools —not *in spite of* their teachers but *on account of* them.

EDITOR'S NOTE. The reader will note that while Professor Hickerson objects to the early deductive explanation and teaching of the principles of the number system, his explanations tend to use an understanding of these principles. What he is objecting to is the denial of opportunity for children to explore, to think, and to discover these principles and thus depriving them of developing power of insight. But so much of discovery in arithmetic depends upon some understanding of the number system and number relationships inherent in the system and hence it would seem desirable to foster learning something (via thinking, discovery, and perhaps even telling) about the role of the base 10 and of place value. This need not be done in one package and certainly should not be attempted at very early age with most youngsters.

The editor agrees totally with Professor Hickerson's two final paragraphs. The Russian Sputnick has awakened us a little. Certainly our more able pupils can learn more and better arithmetic if we, the teachers, honestly try to teach more and better arithmetic.

Teaching Aids for Arithmetic; Cross-Number Puzzles

```
 1?4      ?45?      A4       AA      THIS442
 ?38       ?14      1A       BB       IS1717
----     ----     ----     ----     -------
 73?      8170      ??      1A2     323EASY

  7?       ??      ABB      AB7       ??
   8        7        3        3       3?
----     ----     ----     ----     ----
 5?4      ???1     ?A?1     BBA      1??
                                     ?6
                                    ----
 7)???    A)6?4                      ???
    4        7A
```

A/? + A/?A = 60/91

Students from seven to eighty derive an "exhilerating sense of mental prowess" from the activity of replacing each question mark and letter by a digit that fits, for whatever operation is compatible.

Write for free samples, mentioning the grades and arithmetical operations in which you are interested.

RALPH D. DONER
2305 Gill St., S.E., Huntsville, Alabama

Essential Mathematical Meanings in Arithmetic

in Quantitative Thinking, Computation, and Problem Solving

E. T. McSwain
Northwestern University, Evanston, Ill.

Ralph J. Cooke
Fond du Lac, Wisconsin

"Arithmetic is the foundation of all mathematics. It is the most useful of sciences, and there is probably no other branch of human knowledge which is so widely spread among the masses."

Tobias Dantzig

Psychological Concepts Involved in Teaching Arithmetic

1. The meaning of numbers and the meanings involved in computation must be discovered and learned by each pupil. These meanings then exist in the minds of the pupils.

2. Teachers may find it helpful to remember that each pupil discovers, interprets, and develops his own understanding of the meanings, computation operations, vocabulary, and arithmetical signs.

3. Teachers should provide instruction, illustrations, activities, and examples which will help each pupil to learn the meanings, computation operations, vocabulary, and arithmetical signs so that his mental system of arithmetic will be consistent with the Hindu-Arabic or decimal number system, operations, and vocabulary.

4. Teachers should remember that all computation in arithmetic is mental. The writing of an example and the steps in computation serve as an aid to pupils in mental thinking and computation. A recorded answer shows or notates the result of a pupil's mental process used in finding the answer to an arithmetical example or a problem involving the use of numbers.

5. Reasonable is the view that the difficulties experienced by many pupils in learning and using the fundamental operations with integers, common fractions, decimals fractions, or denominate numbers are the result of faulty or rote learning. The difficulties are in the mind of the pupil rather than in the decimal number system and operations.

6. Research supports the view that each pupil can learn only at his unique "rate of learning" pattern. It is normal for pupils to be different in their learning and use of arithmetic.

7. It is recommended that teachers recognize that understanding and quality of thinking are more important in the work of pupils than mere quantity of examples or problems worked.

8. How each pupil has learned to interpret the meanings, mental operations in computation, and vocabulary affects his interest and ability in applied arithmetic.

9. Rote learning of rules, vocabulary, and operations can easily prevent many pupils from discovering and developing proper meanings, thought processes, and vocabulary involved in correct thinking and computation in arithmetic.

10. Teachers, after they have evidence that pupils understand properly the meanings, operations, and vocabulary they have taught should provide various kinds of practice to help pupils remember what has been learned.

11. Teachers may guide more effectively the learning on the part of pupils when;they interpret the mental processes involved from: (a) awareness of concrete objects, (b) mental perception of the amount and/or relationship of the objects, (c) the arithmetic name to be applied, (d) the use of a number to identify the perceived amount, (e) the writing or notating of the number. (All notated numbers stand for arithmetical thinking.)

12. Teachers who understand the meanings, computation operations, and vocabulary in arithmetic experience greater interest and success in helping pupils to learn and build a mental system of arithmetic consistent with the system that is universally used in industry and business.

Curriculums in Arithmetic in the Classroom

1. It is important that teachers examine their understanding of the three related and also different curriculums in arithmetic that are in each classroom.

a. There is the "teacher's" curriculum. It consists of the meanings, computation operations, vocabulary, objectives and so forth, that the teacher understands and desires to help pupils learn.

b. There is the "textbook" curriculum. It offers a sequential presentation of the meanings, operations, and terms which are in arithmetic and provides in addition, questions, explanations, illustrations, examples, and problems as resource material in helping pupils to understand and use with meaning and accuracy mental processes in applied arithmetic.

c. There is the "pupil's" curriculum. The pupil's "under-the-skin" discovery, interpretation, and understanding of arithmetical meanings, operations, and vocabulary comprise his curriculum. Each pupil through observing, thinking, and application builds his psychological curriculum in arithmetic.

2. In each grade there is a need to plan for the different kinds of curriculums.

a. There must be a teacher-planned curriculum based on desired outcomes for pupils.

b. There must be a teacher and pupil cooperatively-planned curriculum.

c. There must be time for instruction periods when pupils are motivated to discover and learn, under teacher guidance, new meanings and operations and their relation to previous leanings.

d. There must be time for teacher and pupils to discuss and evaluate what has been learned in arithmetic.

An Interpretation of Arithmetic

1. Arithmetic is a man-made language with a number system that is used to identify, record, and communicate numerical ideas of the amount of quantity and different relationships of quantities.

2. Arithmetic is the science of numbers and is the foundation of all sciences.

3. The Hindu-Arabic system of notation and computation is universally adopted. The system is based on place-value positions of 1, 10, and powers of 10.

4. The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are symbols to notate (to show) the frequency of units (ones) in any of the place-value positions. (475 or 4 hundreds, 7 tens, 5 ones).

5. The symbol, 0 (zero), at the lower levels, is used to show the absence of a unit in a given place-value position.

6. Notated numbers represent the result of a mathematical and mental process in organizing and stating the amount of quantity in condensed numerical form. (One hundred books are notated by the number 100 which represents an idea of 1 hundred, no tens, no ones).

Foundational Meanings in Arithmetic (Elementary School Mathematics)

Numbers and the Number System

1. There is a one-to-one relationship between quantity, conceptual interpretation of amount, and notated number.

2. A cardinal number expresses the numerical amount of a given group of objects. (37 books)

3. An ordinal number expresses the position of an object or number in an arranged numerical series. (Third seat in row 5)

4. The Hindu-Arabic system of notation is based on place-value positions of 1, 10, and powers of 10.

5. The duodecimal system of notation is based on place-value positions of 1, 12, and powers of 12.

6. In the decimal or Hindu-Arabic number system, the mathematical symbols or digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are used to show the frequency of units in each place-value position of a number. (7, 38, 543, 20, 476)

7. The maximum number of units to be shown or notated in any given place-value position is 9. The smallest natural number of units to be shown or notated in any given place-value position is 1.

8. The zero, 0, is a symbol that is used to show the absence of units, or a digit, in any place-value position.

9. Counting is a mathematical and mental process of adding. (1+1=2, 2+1=3, 3+1=4, etc.)

10. A value of ten in any place-value position must be notated in the decimal number system by using 1, in the next higher place-value position.

11. Large numbers are notated in an order pattern of three place-value positions. (2,465; 578, 403; 387, 042, 637; etc.) The first order or period is called ones, the second is called thousands, and the third is called millions, etc.

Addition

1. Addition is a mathematical and mental process of adding numbers and expressing the total by one notated number.

2. Only digits in like place-value positions can be added.

3. There are 81 basic combination facts in addition; 36 with a sum less than 10, and 45 with a sum 10 or more.

4. There can be no zero combination facts in addition.

5. When the total of the digits in any place-value position equals ten or more, substitute for each ten a one in the next higher place-value position.

6. Some of the developmental levels in mental addition are:

4	6	14	36	24	54	84
+3	+9	+ 5	+ 8	+33	+28	+79

7. The meaning and thought processes involved in addition are similar for integers, common fractions, decimal fractions, and mixed decimals.

a. Only digits in like place-value positions can be added.

b. For common fractions, the denominator shows the kind of fractional units. Only like fractional units (fractions with like denominators) can be added.

c. The sum is the number that expresses the amount of all the addends when added.

8. Representative addition examples follow

6	68	$\frac{6}{8}$		$\frac{6}{4}$ or $\frac{3}{2}$		.6	.66	.68
+3	+36	$+\frac{3}{8}$		$+\frac{3}{2}$	$\frac{6}{4}$	+.3	+.33	+.36
9	104	$\frac{9}{8}$ or $1\frac{1}{8}$			$\frac{12}{4}$ or 3	.9	.99	1.04

Subtraction

1. Subtraction is a mathematical and mental process to find the difference between two notated numbers.

2. The idea of "take away" or "removal" in subtraction can apply only to concrete materials.

3. In subtraction of notated numbers, the mental and arithmetical process is to find the difference between the subtrahend and the minuend.

4. The basic combination facts in addition can be used to find the numerical difference between the minuend and the subtrahend.

5. When a digit in any place-value position in the minuend is smaller than the digit in the same place-value position in the subtrahend, the numerical order of the minuend must be changed by substituting for one unit in the next higher place-value position ten units in the next lower place-value position. (32−17=? becomes 2 tens and 12 ones−1 ten and 7 ones)

6. The meanings and thought processes involved in subtraction are similar for integers, common fractions, decimal fractions, and mixed decimals.

7. Representative subtraction examples follow.

8	$\frac{8}{12}$	.8	.88	7 14. 7 14 8 4. 8 4
−6	$-\frac{6}{12}$	−.6	−.66	−2 6. 2 6
2	$\frac{2}{12}$ or $\frac{1}{6}$	.2	.22	5 8. 5 8

Multiplication

1. Multiplication is a rapid, mental process of adding a given number of like-value numbers. (3×8=24) (3×$\frac{7}{8}$=?) (3×.8=?)

2. The meanings and thought processes in multiplication are similar for integers, common fractions, decimal fractions, and mixed decimals.

3. The multiplier, when an integer, shows the number of like-value numbers to be added or multiplied. ($\overline{8}\times 7=56$)

4. The multiplicand shows the numerical value of each of the like-value numbers to be combined or multiplied. ($7\times\overline{6}=42$)

5. In the decimal number system, pupils may experience greater accuracy in multiplication when they understand the following principles:

ones × ones = ones
ones × tens = tens
ones × hundreds = hundreds

tens × ones = tens
tens × tens = hundreds
tens × hundreds = thousands

ones × hundreds = hundreds
tens × hundreds = thousands

hundreds × hundreds = ten thousands

6. When the multiplier is a common fraction or a decimal fraction, the product will be a number smaller in numerical value than the number that is the multiplicand.

7. Representative multiplication examples follow.

$$\begin{array}{r} 6 \\ \times 4 \\ \hline 24 \end{array}$$

$$\times \begin{array}{|c|c|c|c|} \hline & \text{TH} & \text{H} & \text{T} & \text{O} \\ \hline & & & 6 & 6 \\ & & & 4 & 4 \\ \hline & & & 2 & 4 \\ & & 2 & 4 & \\ & & 2 & 4 & \\ & 2 & 4 & & \\ \hline & 2 & 9 & 0 & 4 \\ \hline \end{array}$$

$$\begin{array}{r} \frac{4}{8} \\ \times 4 \\ \hline \frac{2\,4}{8} \text{ or } 3 \end{array} \qquad \begin{array}{r} .6 \\ \times 4. \\ \hline 2.4 \end{array} \qquad \begin{array}{r} .06 \\ \times 4. \\ \hline .24 \end{array} \qquad \begin{array}{r} .66 \\ \times 4. \\ \hline 0.24 \\ 2.4 \\ \hline 2.64 \end{array}$$

$\frac{1}{2} \times 9 = 4\frac{1}{2}$ $\qquad$ $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$(\frac{1}{2} \times \frac{3}{4} = \frac{1}{2}$ of $(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8})$

$$\begin{array}{r} 4. \\ \times\ .2 \\ \hline .8 \end{array} \qquad \begin{array}{r} 24. \\ \times .6 \\ \hline 2.4 \\ 12. \\ \hline 14.4 \end{array} \qquad \begin{array}{r} .4 \\ \times .2 \\ \hline .08 \end{array} \qquad \begin{array}{r} .04 \\ \times .2 \\ \hline .008 \end{array}$$

$$\times \begin{array}{|c|c|c|c|} \hline & \text{t} & \text{h} & \text{th} & \text{tth} \\ \hline & .1 & 4 & & \\ & .7 & 2 & & \\ \hline & .0 & 0 & 0 & 8 \\ & .0 & 0 & 2 & \\ & .0 & 2 & 8 & \\ & .0 & 7 & & \\ \hline & .1 & 0 & 0 & 8 \\ \hline \end{array}$$

Division

1. Division with written or notated numbers is a mental and mathematical process of finding one number that tells how many numbers like the divisor equal the dividend.

2. Division as a physical act may involve:

a. Dividing a quantity into a given number of equal size units.

b. Finding how many units in an equal group when the quantity is changed into a given number of equal size groups.

3. Division of written or notated numbers presents the questions, "How many numbers can be made or thought to be made when changing the dividend to numbers like the divisor? $24 \div 8 = ?$ or ? 8's = 24. 3 8's = ?"

4. In notated division, the quotient shows how many groups of numbers there will be when the dividend is changed or is thought to be changed into numbers like the number that is the divisor.

5. Multiplication is the most functional process to use when finding the quotient in division.

6. The following illustration shows the long method that must be used when the idea or process of subtraction is used to find the quotient in division.

$$\begin{array}{r} 7 \\ 3\overline{)21} \\ -3 \\ \hline \end{array}$$

$18 - 3 = 15 - 3 = 12 - 3 = 9 - 3 = 6 - 3 = 3 - 3 = 0$

7. Representative division examples follow.

$$\begin{array}{lllll} 7 & 3 & 53 & ? & 2 \\ 3\overline{)21} & 7\overline{)21} & 8\overline{)424} & 36\overline{)72} \text{ or } & 36\overline{)72} \end{array}$$

$$\begin{array}{r} 26 \text{ r } 7 \\ 18\overline{)475} \\ 36 \\ \hline 115 \\ 108 \\ \hline 7 \end{array} \qquad \begin{array}{r} 21\ 95. \\ .04\overline{)87.80} \\ 8 \\ \hline 7 \\ 4 \\ \hline 3\ 8 \\ 3\ 6 \\ \hline 20 \\ 20 \end{array}$$

$$\begin{array}{r} 7.\text{ r } 2 \\ .5\overline{)3.7} \end{array}$$

$2\frac{3}{4} \div \frac{1}{2} = \frac{11}{4} \div \frac{2}{4} = \frac{11}{2}$ or $5\frac{1}{2}$.
($2\frac{3}{4}$ equals $5\frac{1}{2}$ groups of $\frac{1}{2}$ each)
($5\frac{1}{2} \times \frac{1}{2} = \frac{11}{2} \times \frac{1}{2} = \frac{11}{4} = 2\frac{3}{4}$)

8. In division, a different meaning and thought process is involved when the dividend is a number smaller than the divisor.

a. The question to answer is, "What part of the divisor equals the quotient?"

b. The quotient will always be less than 1. (It will be a decimal or a common fraction.)

c. Illustrative examples follow:

$$\begin{array}{r} .5 \\ 48\overline{)24.} \\ 24.0 \\ \hline \end{array}$$

The question asked by the example is, "What decimal part of 48 equals 24?" The mental process is, ".5 of 48 equals 24."

$$1\tfrac{3}{4} \div 2\tfrac{1}{2} = \tfrac{7}{4} \div \tfrac{5}{2} = \tfrac{7}{4} \div \tfrac{10}{4} = \tfrac{7}{10}$$

The question asked, "What fractional part of $2\frac{1}{2}$ equals $1\frac{3}{4}$?" The mental process is, "$\frac{7}{10}$ of $2\frac{1}{2} = 1\frac{3}{4}$."

$$\begin{array}{r} .51 \\ .8\overline{).408} \\ .40 \\ \hline .008 \\ .008 \\ \hline \end{array}$$

The question asked by the example at the left is, "What fractional part of .8 = .408?" The answer is, ".51 or .8 equals 408."

Common Fractions

1. A common fraction is a number.

2. A common fraction is a mathematical symbol used to notate a given number of fractional units or a given number of units in a specific group.

3. The denominator identifies the number of equal divisions made in the base or the total number of iunits. The denominator the name of the fractional units.

4. The numerator identifies the number of fractional units or the number of units in a specific group which are being considered.

5. Only the numerators of *like fractions* can be added, subtracted, multiplied, or divided.

6. Changing a common fraction to larger terms is a mental process of substituting for a smaller number of larger units a greater number of smaller units. $\frac{1}{2} = \frac{2}{4}$. If there are twice as many smaller units, the smaller units will be $\frac{1}{2}$ the size of the larger units. $\frac{1}{2} = \frac{4}{8}$. In $\frac{4}{8}$ there are four times as many units as in $\frac{1}{2}$. Each fractional unit is $\frac{1}{4}$ the size of the larger units.

7. Changing a common fraction to lower terms is a mental process of substituting for smaller units, larger fractional units which are equal in total amount to the given fraction. ($\frac{6}{8} = \frac{3}{4}$)

8. The "measurement size" of a fractional unit depends on the size of a given base unit and the number of equal divisions made in the base unit. In $\frac{3}{4}$ ft. the measurement size of each of the three fractional units is $\frac{1}{4}$ foot or 3 inches.

9. When the ratio between two numbers is expressed in fraction form, a different meaning of a fractional symbol is involved.

The symbol represents the ratio relationship between two numbers.

Decimal Fractions

1. A "pure" decimal fraction notates a unit or units with a numerical value less than 1. (tenths, hundredths, thousandsths, etc.)

2. The decimal point is a symbol used to show separation between the one's place-value position and the tenth's place-value position.

3. The decimal value of each fractional unit is shown by the value of the smallest place-value position. (.38)

4. Only digits in like place-value position can be added, subtracted, multiplied, or divided.

5. A mixed decimal notates an integer and a "pure" decimal fraction. (47.058)

Business Use of Decimals

1. Per cent is a business form to express one or more hundredths. The symbol, %, identifies hundredths.

2. The three applications of per cent and percentage in business are:

a. What is a given per cent of a given number? 5%×400.=?

b. A given number is what per cent of another given number? 12 is what % of 24?

c. A given number is a given per cent of what number? 8 is 20% of what number?

3. Understanding of profit, loss, discount, interest, account statements, taxes, etc. is essential in functional citizenship.

Measurement and Measures

1. Measurement is a mathematical and mental operation in applying a selected unit of measure to a given quantity.

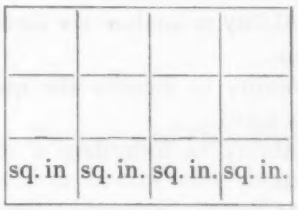
2. An understanding of precision in measurement is essential.

3. The two systems of measures are the English-United States system, and the metric system.

4. The metric system is a scientifically developed system of measures. The three basic units of measure are: meter, liter, gram.

5. An understanding of the measurement system in measuring and recording time is essential. Time should be understood as movement.

6. To find area, a square unit must be used. In the following illustration the question is, "What is the area?" The answer is, "3 rows of 4 sq. in. is the area."



7. To find the volume, a cubic unit must be selected and applied. To find the cubic volume of a box 4 inches long, 3 inches wide, and 5 inches deep, think, "One layer is 3×4 cubic inches. Five layers of 12 cubic inches=60 cu. in. 5×3×4 cu. in.=60 cu. in."

Interpretation of Graphs

1. Ability to read and correctly interpret graphic data as presented by a bar graph, by a line graph, and by a circle graph is necessary in our economy.

2. Understanding of such statistical terms as distribution, mean, median, and mode is helpful in interpreting data.

Geometric Concepts

1. The meaning of straight lines, curved lines, and broken lines must be taught.

2. Ability to ready and understand angles is necessary.

3. Ability to work with squares, rectangles, circles and other geometric figures must be developed.

Formulas

1. Interpretation and use of the most frequently used formulas are necessary.

2. Commonly used formulas include: $A=lw$, $A=bh$, $I=prt$, $V=lwh$.

Drawings and Maps

1. Ability to read and understand scale drawings is necessary.

2. Ability to interpret practical drawings and maps must be developed.

Problem Solving

1. The following abilities are used in solving problems.

a. Ability to read to interpret and define the problem situation.

b. Ability to analyze the facts given in a problem.

c. Ability to identify the question or questions asked.

d. Ability to formulate a reasonable hypothesis.

e. Ability to estimate a reasonable answer.

f. Ability to compute to verify the hypothesis and answer.

g. Ability to derive social and economic generalization from practical problems.

2. The ability to solve many kinds of problems must be developed.

EDITOR'S NOTE. Messrs. McSwain and Cooke have classified many of the things that pupils should understand as they are learning arithmetic. Readers may not find the list inclusive of all of the items they would like and they may quarrel with some of of the statements. But these authors have made a rather complete catalog of concepts and ideas which, if understood, will result in real meaning and understanding of the ideas and operations of arithmetic. How can such a listing be used? Teachers of intermediate grades might well refresh their own understandings and this might be done in discussion groups where teachers exchange ideas on what to do and how to do it. The method of learning employed by pupils usually stems from the concept of arithmetic held by the teacher and hence she should be alert to the thinking involved by pupils as they, as individuals, pursue the development of ideas.

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Thought Processes in Grade Six Problems

CLYDE G. CORLE
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INTRODUCTION

ARITHMETIC, THE "THIRD R" of the elementary school curriculum, is receiving a generous share of attention, both from professional education groups and from laymen as well. Educational leaders vary in their feelings toward the subject. Some show a determination to improve the subject; others are openly critical; and still others seem indifferent to the kind of teaching that is being done. Laymen are often outspoken in their criticism, supporting these opinions by random observation, previous personal school experiences, and hearsay. Standardized tests, teacher-made tests, and performance of textbook tasks serve almost entirely as criteria for judgment, both by professional and lay evaluators.

Like so many other elementary school subjects, arithmetic becomes for the pupil and for the teacher a series of routine tasks, usually dominated by a textbook or a workbook. Pupils follow a day-to-day, page-by-page progession through a book, varying little from the order set by its authors. Teachers judge from mechanical performance on tests, completion of homework assignments, and general effectiveness of a pupil, his progress in arithmetical skill development. Pressures of time, class size, and of conflicting interests have prevented teachers from investigating what may be the real problems which confront pupils in learning how to solve arithmetic problems. These problems are: How does a pupil think when he solves a verbal problem? What does a pupil understand about the problem he is attempting to solve?

The Purpose of this Study

The purpose of this study is to investigate several aspects of pupils' problem-solving behavior which may affect their efficiency in achieving correct answers to verbal arithmetic problems. Specifically, the study attempts to determine the importance of concept formation, methods of reasoning, confidence, understanding of vocabulary, and word pronunciation in oral reading to a pupil when he is attempting to find correct answers to verbal arithmetic problems.

The Basic Hypothesis of the Study

This study is based on the hypothesis that certain identifiable procedures are related to and may affect a pupil's ability to achieve a successful solution to a verbal problem in arithmetic. These procedures are concept formation, reasoning, confidence, understanding of vocabulary, and word recognition in oral reading.

The Procedures Used in the Study

Seventy-four pupils enrolled in two sixth grade classes in the State College, Pennsylvania, Area Schools were used in this study. One class of forty pupils was located in the Matternville School, four miles north of the borough of State College. This school serves a rural district and has a high percentage of children from farms and small communities. The other class of thirty-four pupils attended the Nittany Avenue School, located in the borough of State College.

The children attending this school came from almost every neighborhood in State college as well as from the territory surrounding the borough. These classes were selected especially to get a representative sample of children from the area as a whole.

The children were interviewed singly, and during the interview each was asked to solve in turn the following eight problems:

1. Tom weighs 94 pounds, Jim weighs 88 pounds, Pete weighs 105 pounds, and Ted weighs 111 pounds. What is the average weight of the four boys?
2. When I got up in the morning the thermometer read 7 degrees below zero. At noon the temperature was 15 degrees. How much warmer had it become?
3. Harry bought a used bicycle for $7.50. He spent $3.75 for repairs on it. He then sold it for $15. What was his profit on the bicycle?
4. Sally is the cashier at the school cafeteria. If lunches cost 25¢ per child and 287 children ate at the cafeteria on Friday, what should be Sally's receipts for that day?
5. Last week the cannery where Jack's father works was canning peas. In one day the canner canned 9,492 cans of peas. These were placed in cases holding 24 cans each. How many cases did they fill?
6. Mary's father is a bus driver. Last week he made a trip to New York City. When he left home the speedometer read 35408. When he returned home it read 36020. How many miles did he go on the round trip?
7. A store had a sign in the window which read, "All ice skates ¼ off." Nancy decided to buy a pair which had been marked $9.00. How much would she have to pay for the skates?
8. To pass his first class Scout badge. Bob had to swim twice the length of the swimming pool. How far would he have to swim if the pool is 90 feet by 60 feet?

These problems were chosen because they were believed to be typical of those appearing in sixth grade arithmetic content. The problems were printed on individual cards on which space was left for computation and for recording the answer. Pupils were directed to read each problem orally, then reread it to themselves until they had become certain what it meant. Each pupil was then directed to work the problems on the cards.

A tape recorder was used to record the oral reading efforts and all comments made by the pupil and the interviewer. After each problem pupils were asked the following questions:

1. What is your answer?
2. How did you get it?
3. Why did you work it that way?
4. Do you think your answer is right?
5. Why do you think it is (or is not)?

Each of the seventy-four pupils had an opportunity to work all of the eight problems. No assistance or direction was given, and pupils' problem solutions were accepted without comment.

The tape recordings were copied verbatim, immediately after the interviews. These recordings, the numerical solutions, and the observed behavior of the children were evaluated in the following manner:

1. Was the solution to the problem right or wrong?
2. Was the solution to the problem based on a good, fair, or poor concept of what the problem meant?
3. Did the pupil give evidence that he was applying real or vicarious experiences to his solution; was the solution a mechanical act which was prompted by word or number clues or did he refuse to try to work the problem at all?
4. Was the confidence level of the pupil high, medium, or low when his solution or his answer was discussed?
5. Did the pupil have a good, fair, or poor understanding of selected words used in the problem?
6. Was the pupil able to pronounce the words correctly and fluently when he read the problems aloud? These differences were classified good, fair, and poor.

Accuracy was determined by the correctness of the numerical solution, including the proper placing of the decimal point and either verbal or written interpretation of the concrete values represented. Any computational error, unless it was offset by another, rendered the solution incorrect. A problem which a pupil was unable to solve at all was considered wrong.

Concept evaluation was the result of studying the pupil's algorisms, his verbalizations as he solved the problems, and his answers to questions asked by the investigator. A "good" concept was one which indicated that a pupil could likely objectify the solution, could defend it logically, and could reconstruct mentally either the problem situation or one related to it. The solution did not necessarily need to be correct, for errors in computation sometimes oc-

curred. A "fair" concept was one which was partly right, but perhaps vague or stereotyped in its origin. These were often manifested when a pupil had memorized or conditioned a response to a "type" of problem, or used verbal clues to a kind of computational mechanism. Sometimes these answers were correct, but the child could give little or no insight into how or why he solved the problem as he did. A poor concept was one which was obviously incorrect or quite unrelated to the situation. All problems which the pupil refused to try to solve were classified in the poor group. In a few cases, the pupil was able to show the correct answer on his paper, but was utterly unable to report how he arrived at that answer.

Reasoning methods were determined by interpreting pupil's explanations of their own work. A classification of "experiential" reasoning was given when the pupil related himself to some real or vicarious role in solving a problem. For example, if he said in solving Problem 8 which deals with a swimming pool, "I once did what the problem describes" or some other such remark, it was generally classified as "experiential." If, however, the only reference made by the pupil was to number relations, clues, or computation, the solution was said to be "computational." The classification "none" was assigned to problems which the child would not attempt to work.

Confidence levels were established by studying answers to questions of (1) whether or not the pupil believed his answers to be right, and (2) why he thought as he did. "High" confidence was credited when the pupil said with positiveness that he thought his solution was right. "Medium" confidence was credited when some doubt was expressed or when there was reluctance to affirm the correctness of a solution. "Low" confidence was credited when a pupil was very sure he was wrong, or quite unsure he was right. In no case did the accuracy of the solution affect the investigator's appraisal of the pupil's confidence.

Vocabulary ratings were given on interpretations of certain key words in each problem. In some cases these words were adequately interpreted during the solution of the problem. In other cases questions were asked to clarify the puptil's definition of a term. A "good" rating was given when a pupil was able, either through definition or example, to identify what the word meant in that problem. A "fair" rating was given when a pupil used a word in a correct or partially correct sense, but failed to apply his interpretation to the solution of the problem. If more than one word in a problem was defined and any was incorrect, no better than "fair" could be given. "Poor" ratings were given in vocabulary when the pupil could not identify the meaning of key words in the problem, gave incorrect definitions, or used wrong interpretations.

Reading efficiency was judged solely on the fluency of word pronunciation as the pupil read the problem aloud. A "good" classification was assigned when the problem was read orally with fluency and without assistance. The reading was considered "fair" when there was some hesitancy, incorrect pronunciation, and lack of word recognition. In general, the "fair" reader could read the problem orally with limited assistance. "Poor" reading was characterized by labored pronounciation, failure to recognize many words, and halting, irregular oral reading procedures.

Treatment of the Data

The following procedures have been employed in presenting the data of this study:

1. The relationship between accuracy in problem solving and the five problem solving procedures being investigated were analyzed by Chi Square and Coefficient of Contingency techniques. Results of these analyses are shown in Table I.
2. Classification of accuracy in problem solving and of the five problem-solving procedures were summarized for each problem. These summaries are presented in Table II.
3. Differences in problem solving accuracy between the two schools were analyzed and the results of the Chi Square analysis are shown in Table III.

TABLE I

ANALYSIS BY CHI SQUARE AND BY COEFFICIENT OF CONTINGENCY BETWEEN ACCURACY IN PROBLEM SOLVING AND FIVE OTHER CRITERIA FOR EVALUATING PROBLEM SOLVING BEHAVIOR IN SEVENTY-FOUR SIXTH GRADE PUPILS IN THE COLLEGE AREA SCHOOLS

	Chi Square	Degrees of Freedom	Level of Confidence	Contingency Coefficient
Accuracy with concept	153	2	.01	.82
Accuracy with reasoning	8.4	1	.01	.31
Accuracy with confidence	25	2	.01	.50
Accuracy with vocabulary	15.6	2	.01	.41
Accuracy with oral reading	5.6	2	.06	.26

The first step in presenting these data will be to interpret the statistical relationships which seem to exist between the problem-solving accuracy and each of the five problem-solving procedures. The second step will seek to explain and interpret the significance of each of the problem solving procedures, with illustrations of pupil responses related to each. The third step will attempt to show and interpret differences in problem solving accuracy which appeared between the schools.

THE DATA

Chi Square and Coefficient of Contingency Relationships between Problem Solving Accuracy and Five Problem Solving Procedures. Analysis by Chi Square between clarity of

TABLE II

DISTRIBUTION OF ACCURACY, CONCEPT, REASONING, CONFIDENCE, VOCABULARY, AND FLUENCY OF ORAL READING IN EVALUATING EIGHT VERBAL PROBLEMS IN ARITHMETIC SOLVED BY SEVENTY-FOUR SIXTH GRADE PUPILS IN THE STATE COLLEGE, PENNSYLVANIA AREA SCHOOLS

Problem No.	Accuracy		Concept			Reasoning			Confidence			Vocabulary			Reading		
	R	W	G	F	P	C	E	N	H	M	L	G	F	P	G	F	P
1	37	37	19	19	36	69	12	1	48	16	10	14	16	44	55	11	8
2	19	55	15	2	57	63	11	0	50	14	10	54	15	4	55	10	9
3	34	40	36	10	28	55	9	10	46	8	20	41	13	20	54	9	11
4	46	28	33	23	18	71	3	0	58	8	8	36	11	27	53	11	10
5	40	34	24	27	23	72	2	0	53	11	10	59	10	5	55	9	10
6	46	28	32	18	24	68	6	0	59	5	10	58	8	8	54	9	11
7	11	63	10	9	55	60	4	10	37	7	30	19	26	29	56	7	11
8	37	37	33	3	38	48	24	2	52	5	17	43	6	25	56	8	10
Totals	272	320	202	118	272	508	61	73	403	74	115	326	102	164	438	74	80

Symbols: R—Right W—Wrong G—Good F—Fair P—Poor C—Computational E—Experiential N—None H—High M—Medium L—Low

concept and accuracy of problem solving shows a high degree of relationship. This statement is based on a comparison of observed manifestation of concept by seventy-four sixth grade pupils as they solved verbal problems in arithmetic with their problem solving accuracy. Significant to the .01 level of confidence, this relationship leaves little doubt that students are more successful in problem solving when they understand the meanings of the problems.

The methods of reasoning considered in the study were "experiential" and "computational." While a third classification "none" was used in the study, it was impossible to use this category statistically, for obviously, a pupil could not get a problem correct if he did not attempt to solve it. The highest frequency observed was that of computational reasoning. That frequency, when compared with accuracy of problem solving, showed a very high relationship. This, too, was significant at the .01 level of confidence, indicating that pupils may achieve a degree of accuracy using principally word and number clues to problem solving.

A close statistical relationship likewise existed between confidence and accuracy in problem solving. A Chi Square analysis shows that at the .01 level of confidence a belief in a pupil's own effectiveness is closely related to accuracy of problem solving.

Understanding of the vocabulary used in the problem and accuracy were also very closely related to each other. This relationship, also, was shown at the .01 level of confidence. It seems to indicate that the pupil must know and understand the meanings of words used in the verbal problems he attempts to solve.

Only the procedure of oral reading failed to show a high degree of relationship to accuracy in problem solving. The Chi Square of 5.6 is significant at only the .06 level of confidence, leaving some doubt whether fluency in oral reading plays an important part in accuracy in problem solving.

These statements may appear at first to be somewhat contradictory, and perhaps they are. One of the objectives of teaching arithmetic is to achieve abstractness. This abstractness tends to minimize the expression of concepts in detail, and to find short cuts eliminating unnecessary thought processes. Pupils seemed to have clear understanding of what they were trying to do, yet did not seem willing or able to verbalize it for the investigator. At times the problem was worked mechanically, the answer was stated abstractly, and the pupil appeared to be entirely removed from the process. Only through persistent probing could the idea behind the solution be uncovered. This paper does not seek to establish one procedure as more desirable than another, but rather to show the relationship that appeared to exist. Table I shows the statistical results in detail.

Distribution by Problem Showing Accuracy in Relation to the Five Procedural Areas. When shown in somewhat greater detail, the results take on new meaning. Table II shows the distribution of frequencies in each of the five procedural areas in relationship to the accuracy of problem solving.

Number and Percentage of Right and Wrong Answers. Eight problems were attempted by each of the seventy-four pupils during this study. The pupils solved 272, or about 46% of these problems correctly. They solved 294 incorrectly and did not attempt to solve the remaining 23 problems.

The Frequency of Good, Fair, and Poor Concepts by Pupils. Solutions to the 592 problems indicated that 202, or 34% of them, were solved by pupils who had a clear understanding of the meanings of the problems and the processes used in solving them. A good concept has already geen described, and may be illustrated by the work of Joe F. in solving Problem 2.

PROBLEM 2. When I got up in the morning the thermometer read 7 degrees below zero. At noon the temperature was 15 degrees. How much warmer had it become?

There was no oral reading problem. Joe read rapidly and fluently.

*Inv:** What is your answer?
Pupil: 22 degrees.
Inv: How did you get it?
Pupil: Well . . . I added 7 and 15 . . . .
Inv: Why?
Pupil: Well you start out at zero and you go down 7 . . . that makes 7 below . . . and then with the 15 . . . that's 15 above zero . . . and you add 'em together and it comes out 22.
Inv: What do you mean by temperature?
Pupil: Well . . . it's when it's cold or hot . . . .
Inv: It is right?
Pupil: The answer is . . . .
Inv: Why?
Pupil: 'Cause I went over it two times . . . and it doesn't seem like too hard a problem . . . .

Fair concepts, which revealed sufficient understanding to achieve a correct or near correct solution, but left some doubt about actual interpretation of meaning were apparent in 111 or 19% of the solutions. Such a concept might be illustrated in Lois R.'s solution to Problem 5.

PROBLEM 5. Last week the cannery where Jack's father worked was canning peas. In one day the cannery canned 9492 cans of peas. These were placed in cases holding 24 cans each. How many cases did they fill?

There was no oral reading problem. Lois read fluently and correctly.

Inv: What is your answer?
Pupil: 395 and 12 people . . . .
Inv: Do you mean 12 cans?
Pupil: Uh . . . Uhuh . . . .
Inv: Is that a part of a case?
Pupil: Uhuh . . . one-half . . . .
Inv: How did you work it?
Pupil: I divided . . . .
Inv: Why did you divide?
Pupil: I wanted to know the average . . . how many cans . . . were placed in a box . . . they wanted to know . . . how many . . . boxes they filled . . . .
Inv: What is a case?
Pupil: Like a box . . . you put things in . . . .
Inv: Is the problem right?
Pupil: Uhuh . . . .
Inv: Why?
Pupil: Because . . . (long pause) . . . .
Inv: Did you use the right method?
Pupil: (Long pause) . . . uhuh . . . .

Poor concepts, which indicate unfamiliarity with the situation presented by the problem or lack of understanding of the situation was observed in 279 or 47% of the solutions. This type of concept is observed in the work of Marnie S. in solving Problem 3.

PROBLEM 3. Harry bought a used bicycle for $7.50. He spent $3.75 for repairs on it. He then sold it for $15.00. What was his profit on the bicycle?

There was no apparent reading problem. She failed, however, to recognize the word "profit" and pronounced it questioningly.

Inv: What is your answer?
Pupil: $11.25
Inv: How did you get it?
Pupil: I added $7.50 and $3.95 . . . (note error in second number).
Inv: Why did you do that?
Pupil: Well . . . it says . . . what is the profit of the bike . . . and the . . . I think it's right . . . if you'd want to find out altogether how much he'd expect for his bike . . . then you would add . . . .
Inv: What does profit mean?
Pupil: It means . . . what was the total amount . . . .
Inv: The total amount spent for the bike?
Pupil: Yes.
Inv: Is your answer right?
Pupil: Yes.
Inv: Why do you think so?
Pupil: 'Cause I added it right . . . .

Attention is called to the fact that only 202 "good" concepts were noted, yet 272 problems were solved correctly. This difference indicates that as many as one out of four problems was correctly solved without a clear concept of the actual meaning of the problem.

The Nature of the Reasoning Process. Almost all of the reasoning processes used by the pupils during this investigation were computational in nature. In 508 out of 592 cases the method for working the problem was suggested by number or vocabulary clues, configuration of the problem, or the nature of the question asked. In only 61 solutions, or about ten per cent, was there any reference to a real or vicarious experience which the pupil associated with the solution of the problem. One example of "computational" reasoning may be seen in the work of a girl, Shirley R., as she worked Problem 1.

PROBLEM 1. Tom weighed 94 pounds, Jim weighed 88 pounds, Pete weighed 105 pounds, and Ted weighed 111 pounds. What is the average weight of the four boys?

There was no reading problem apparent. The words were all correctly pronounced.

Inv: What is your answer?
Pupil: 99 . . . 2R.
Inv: What do you mean . . . 2R?
Pupil: One-half . . . .
Inv: How did you get it?
Pupil: I added how much each boy weighs . . . and I took 4 into it . . . and I got 99½ . . . .
Inv: Why did you do that?
Pupil: 'Cause average means . . . whenever you have numbers like that . . . you're supposed to add and then divide . . . .

* Investigator.

Inv: What do you mean by "average?"
Pupil: Average is . . . (pause) . . . . It's add and then divide . . . .
Inv: Is it right?
Pupil: I don't know . . . .
Inv: Might it be?
Pupil: Yes . . . .
Inv: Why?
Pupil: 'Cause you're supposed to add . . . and then divide . . . .

Jack F.'s solution to Problem 8 is representative of the experiential kind of reasoning mentioned in the preceding paragraph.

PROBLEM 8. To pass his first class swimming badge, Bob had to swim twice the length of the swimming pool. How far would he have to swim if the pool is 90 feet by 60 feet?

There was no problem of word recognition or of pronunciation.

Inv: What is your answer?
Pupil: 180 feet.
Inv: How did you get it?
Pupil: Well . . . it's the length of the pool . . . and the length is the longest way . . . it didn't say he had to swim all the way around it two times . . . just said he had to swim the length . . . two times . . . and the length is 90 feet . . . I multiplied 2 times 90.
Inv: Is your answer right?
Pupil: Yes.
Inv: Why?
Pupil: Well . . . 2 times 9 is 18 . . . 2 times 0 is 0 . . . .

Pupil Confidence in Problem Solving Results. Answers to the questions, "Do you think your answer is right?" and "Why do you think so?" indicate that in 403 cases, or about 68% of the total, the pupils were sure that their answers were right. In 74 cases, or about 13%, there was some expression of doubt. The pupils were sure they were wrong in 115 cases, or 19% of the total. It is noticeable that confidence outweighs accuracy by almost 50%. It is likewise of note that pupils were unable to identify only about one out of three incorrect solutions.

Vocabulary as A Factor in Problem Solving. A "good" understanding of the words used in the problems occurred in 326, or 55% of the solutions. This means that the pupil appeared to be familiar with the key words used in the problem statement and was able to interpret them correctly, either as he discussed the question or when the investigator asked specific questions about the words. The solution to Problem 7 by Ralph A. is typical of casual, but correct, interpretation of the vocabulary of a problems.

PROBLEM 7. A store had a sign in the window which read, "All ice skates ¼ off." Nancy decided to buy a pair which had been marked $9.00. How much would she have to pay for the skates?

The problem was read with no mispronunciation of words.

Inv: What is your answer?
Pupil: $6.75.
Inv: How did you work it?
Pupil: I . . . uh . . . multiplied ¼ times $9.00 . . . that she would have had to pay for it . . . and got $2.25 . . . that was supposed to be taken off the price . . . and I subtracted that from $9.00 . . . and got $6.75.
Inv: Is you answer right?
Pupil: Yes.
Inv: Why do you think so?
Pupil: I'm pretty sure that's the way you do it . . . and that's the way I've always done it . . . and I got it right.

The above solution also reflects experiential reasoning, and high confidence in the accuracy of the solution.

In spite of a "fair" understanding of the vocabulary, Margaret L. presented an incorrect solution to Problem 6.

PROBLEM 6. Mary's father is a bus driver. Last week he made a trip to New York City. When he left home the speedometer read 35408. When he returned home it read 36020. How many miles did he go on the round trip?

This pupil had trouble reading the five-digit numbers. She read them "thirty-five thous . . . no . . . three million, five thousand, four hundred and eight." Her reading was otherwise reasonably fluent and correct.

Inv: What is your answer?
Pupil: Seven million, one thousand, four hundred, twenty-eight . . . (71428).
Inv: How did you work it?
Pupil: I added 3 million . . . etc.
Inv: Why did you do that?
Pupil: Well . . . I guess if you added the miles that he started with . . . and the miles when he gotted there . . . that would be how many miles he rode . . . .
Inv: What is a speedometer?
Pupil: It's a thing that tells how many miles . . . .
Inv: What is a "round trip?"
Pupil: Well . . . if somebody would go . . . uh . . . from here at State College . . . to . . . uh . . . Washington . . . and coming back then . . . .
Inv: Is it right?
Pupil: No.
Inv: What is wrong?
Pupil: I don't know . . . .
Inv: Can you work it another way?
Pupil: Huh . . . uh . . . I guess it's right . . . .

Poor interpretation of vocabulary was apparent in this solution by Kay C. Not only did she make the incorrect interpretation of the word "receipt" but it caused her to get the problem wrong after she had already worked it correctly. Below is Kay's solution to Problem 4.

PROBLEM 4. Sally is the cashier at the school cafeteria. If lunches cost 25¢ each and 287 children ate at the cafeteria on Friday, what should be Sally's receipts for that day?

The pupil read all of the words fluently, without any mispronunciation. She worked the problem by multiplication and got the correct answer. She appeared dissatisfied with this answer and then put down 287 divided by 25. After trying for a minute to divide this she erased her division and put down 287 as her answer.

Inv: What is your answer?
Pupil: 287.
Inv: How did you get it?
Pupil: By just thinking . . . .
Inv: How did you think?
Pupil: Well . . . how many receipts there was . . . there . . . .
Inv: What do you mean?
Pupil: Each child would have a piece of paper . . . or something . . . .
Inv: Why?
Pupil: Well . . . if 287 children ate at the cafeteria . . . and that would mean there would have to be 287 slips . . . .
Inv: Is your answer right?
Pupil: Yes.
Inv: Why?
Pupil: Well . . . (mumble) . . . twenty-eight . . . children . . . there'd have to be 287 slips there . . . .

Fair interpretation of vocabulary occurred in 102 solutions, or 18% of the total number. The interpretation "fair" was given chiefly when questionable definitions were given or when the definitions were inconsistent with the solution to the problem. Poor or distorted understanding of critical words occurred in 164, or 26% of the problems solved.

Fluency in Word Pronunciation in Oral Reading. The oral reading which preceded each pupil's attempt to solve his problems showed that most of the pupils were able to pronounce all or nearly all of the words fluently and correctly. There was much consistency in this respect, for "good readers" appeared to read all of their problems well, "fair readers" not so well, and "poor readers" with a minimum of variation, all poorly. Fifty-five pupils read 440 problems reasonably well. Nine pupils read 72 problems fairly well, and 10 pupils read 80 problems poorly.

The ability to read problems well and to interpret the vocabulary correctly was somewhat better than the accuracy in problem solving.

The Frequency of Incorrect Computation. Mistakes in computation occurred 114 times in the 592 solutions, or in about 19% of the total. These errors were noted whether or not the computation was appropriate to the solution to the problem. In other words, if the pupil used the incorrect method of working the problem and made no error in his work, it was considered right, as far as computational accuracy was concerned. Of the 114 errors noted, answers to 70 problems only were affected. The other 44 errors were made in doing inappropriate computation, and the pupil could not have achieved a correct solution if there had been no error.

Analysis by Individual Problem. Specific difficulties were encountered by pupils in solving certain of the eight problems. Table II has outlined by problem the various procedural areas and has given the frequency of each classification. The problem which seemed hardest to solve correctly was number 7, a two-step problem which asked the pupils to find the discount on and the selling price of a pair of skates. Only eleven pupils worked the problem correctly. Ten pupils appeared to have a "good" concept and nine were considered to show a "fair" concept. Sixty of the pupils used computational reasoning, and all but nineteen showed vocabulary difficulty. Only four seemed able to draw on their own experiences in solving and explaining this problem.

The easiest of the problems were numbers 4 and 6, both of which were solved correctly by 46 pupils. Problem 4 required multiplication of a three-digit number by a two-digit number. Problem 6 called for subtraction of two five-digit numbers. In both problems, however, the concept level was about

25% below the accuracy level. The confidence level was aprpoximately 25% above accuracy levels in both problems. Some differences were seen, however, in vocabulary interpretation. In Problem 4, the number of "good" interpretations was about 25% below the accuracy count, and in Problem 6 it was 25% above the accuracy level.

Half of the pupils worked Problem 1 correctly. This problem asked the pupils to find the average of four numbers. Only nineteen pupils showed a "good" concept of the meaning of the problem, and all but fourteen had trouble in interpreting the vocabulary used in the problem. Sixty-one of the pupils used some form of computational reasoning, and only twelve were able to relate the ideas of the problem in some way to their own experiences.

Problem 8 dealt with the measurements of a swimming pool. Like Problem 1, half of the pupils worked it correctly. "Good" concepts, however, were more frequent in this problem that in Problem 1. There were 33 examples of "good" unterstanding of the problem. Twenty-four pupils referred to something in their own experiences in explaining their solutions. Both the confidence levels and the vocabulary interpretations were somewhat above the accuracy level for Problem 8.

Comparison of Performances of Pupils Between Schools. Analysis by Chi Square was made of the right and wrong answers in the two schools used in the study. The results of this analysis are shown in Table III. Only two of the problems showed significant differences, Problems 1 and 8. These differences were significant at the .01 level of confidence. In both cases, the Nittany Avenue School showed superiority over the Matternville School. In Problem 1, there was a total of nineteen "good" concepts. Of this total, thirteen were found in the Nittany Avenue School. Of the fourteen "good" vocabulary interpretations, 10 were in the group which showed superior accuracy in solving the problem.

The Nittany Avenue pupils, likewise, did better on Problem 8. This problem required computation with linear measurements of a swimming pool. Twenty of the 33 "good" concepts found in studying the problem solutions appeared in the work of the Nittany Avenue pupils. Thirty of the 43 "good" vocabulary interpretations were given by this class. It might be said as an item of interest that the State College community has both an indoor and an outdoor swimming pool, patronized to a considerable extent by children. The Matternville community has little access to this kind of recreation.

SUMMARY AND CONCLUSIONS

Summary of the Data. The following statements represent the significant aspects of the data:

1. Analysis by Chi Square and the coefficient of Contingency shows a high degree of relationship (at the .01 level of confidence) between accuracy in problem solving and "good" concept, computational reasoning, "high" confidence, and "good" vocabulary interpretation. Significant at the .06 level of confidence was the relationship with fluency in word pronunciation in oral reading at the "good" level.
2. Seventy-four pupils worked or attempted to work 582 problems. Of this number 272, or 34% of the total, were correct. The pupils solved 294 incorrectly and failed to work 23 of them.
2. "Good" concepts were evident in 202 of the solutions, "fair" concepts in 111, and "poor" concepts in 279 solutions.
4. In 508 cases, the reasoning appeared to be computational in nature. Only in 61 cases did pupils give reasons or explanations which seemed to be related to their own experiences.
5. Pupils believed their answers to be right in 403 of the solutions. They were doubtful about 74 answers, and they were sure that 115 were wrong.
6. A good understanding of vocabulary was observed in 326, or somewhat more than half of the problem solutions. "Fair" interpretation occurred in 102 problems, and "poor" or distorted interpretations of terms 164 times.
7. All except nineteen pupils read fluently and pronounced the words well. Nine were considered "fair" readers, and ten were labeled "poor" readers. The good readers read 440 problems well.
8. There were 114 instances of incorrect computation in solving the 592 problems. Seventy of these errors occurred when the method selected was right, and the computational

error prevented a correct solution of the problem. Forty-four of the computational errors did not affect the final outcome of the solution because the method was incorrectly chosen.

TABLE III

ANALYSIS BY CHI SQUARE BETWEEN ACCURACY OF MATTERNVILLE SCHOOL PUPILS AND NITTANY AVENUE SCHOOL PUPILS ON EACH OF EIGHT VERBAL PROBLEMS IN ARITHMETIC

Problem No.	Chi Square	Degrees of Freedom	Confidence Level
1	6.7	1	.01
2	1.5	1	.21
3	0.4	1	.80
4	1.0	1	.30
5	0.1	1	.85
6	2.0	1	.17
7	0.4	1	.50
8	7.8	1	.01

9. The "hardest" problem, or the one missed most frequently, was a two-step problem involving discount and selling price. Eleven pupils worked it correctly. There were ten "good" concepts and nine "fair" concepts. Nineteen pupils seemed familiar with the words used in the problem.
10. The easiest problems, or the ones missed least often were two in number. Both were one-step problems, one using multiplication and the other subtraction. In these problems the "good" concepts were lower than the accuracy level by about 25%. High confidence exceeded the accuracy level by about the same per cent. Vocabulary interpretation on one problem was higher than and on the other problem lower than accuracy, by about 25%.
11. Only two problems seemed to show any differences between the two schools, Problems 1 and 8. Differences between the schools on these problems, however, were significant at the .01 confidence level. On both problems, the school in State College showed superiority to the rural school. In both cases, the factors which showed a higher degree of under standing were present in a greater degree in the schools which made the highest scores.

Conclusions

Conclusions from the Data. The following conclusions seem to be supported by the data already presented:

1. Good concepts are related to efficiency in problem solving. It is evident that clarity of understanding is an important factor in problem solving accuracy.
2. Computational reasoning is an effective procedure in problem solving. The appraisal of verbal and number clues and application of appropriate computational techniques are significant factors in securing correct solutions to problems.
3. High confidence and problem solving efficiency have a significant relationship to each other. Pupils who get answers correct generally believe in their own accuracy.
4. Ability to interpret vocabulary correctly is related to problem solving efficiency. Pupils who understand the words in the problems they solve are more likely to get these problems right.
5. Correct solutions were about 35% more numerous than "good" concepts. This is evidence that pupils frequently get problems right without any real understanding of the meanings of the problems.
6. Pupils make little reference to life applications in solving or in explaining their problems. Their responses are controlled by verbal and number clues, and the meanings seem to have little significance in the selection of a computational process. This may be abstractness, and it is possible that questioning can produce support of concrete nature for the abstractions used.
7. Pupils tend to be overconfident of their own problem-solving ability. There is evidence that these pupils placed a great deal of reliance on their computational skill. When no error could be found in their algorism, their confidence increased.
8. Poor interpretation of vocabulary may result in incorrect solutions to problems. Sometimes they cause pupils to reject a solution which had been based on verbal or number clues and attempt one appropriate to his new definition of the terms of the problem.
9. There is no significant relationship between oral reading fluency and ability to work problems correctly. Some relationship is apparent, but it is not significant.
10. Computational accuracy is not a major factor in poor problem solving. While it is not entirely unimportant, computational errors caused only twelve per cent of the problems to be missed.
11. The "hardest" problem showed low frequency in each of the desirable areas of problem solving behavior.
12. The "easiest" problem showed higher frequencies in each of the desirable areas of problem solving behavior.
13. No apparent reasons could be found for differences between State College and rural children in their accuracy on Problem 1. It is evident, however, that on all areas of desirable problem-solving behavior the borough children did better than the rural children.
14. Familiarity with swimming pools and ability to visualize such a pool might have been a factor in the success of borough children over rural children in solving Problem 8.

Implications for Teachers.

1. A pupil's idea of what a problem means is important to him. A "good" concept is closely related to problem solving accuracy. It must be remembered, however, that pupils were able to work correctly one out of four verbal problems with no apparent understanding of what the problems meant.
2. Word and number clues served as the predominant method of attack in problem solving. More than five out of six problems were approached in this manner. Teachers must challenge problem solutions more often, use extraneous materials, and compel students to justify methods by concrete examples.
3. Pupils tend to be overconfident about their arithmetical success. This undue assurance indicates over-dependence upon computation as a tool for problem solving.
4. Understanding the terms used in arithmetic is a definite factor in problem solving efficiency. Teachers must insure familiarity with the vocabulary of verbal problems if effective problem solving is to result.
5. The greatest number of incorrect solutions occurred in problems where the method of solution was not apparent, but was dependent upon deductions based on experiences with the idea. Such experiences with ideas are necessary for effective problem solving.
6. Real reasons for missing most of the problems were not computational ones. This fact suggests that undue stress may be placed on mechanical drill materials.

EDITOR'S NOTE. Dr. Corle has sought to investigate and classify the elements that enter into thinking and understanding in problem solving in arithmetic. This is not an easy task because so many elements enter into a mental process within a group of pupils who have different backgrounds of experience, of ability, of previous amounts and methods of learning, and different attitudes toward learning. It may be surprising that Dr. Corle found that clues of words and numbers served as the predominant method of attack in problem solving. Was this because of the previous teaching or in spite of it? He also found that great reliance for confidence was placed upon skill in computations. All of his work tends to support the thesis that problem solving is not a "skill" or an ability that may be acquired by a mechanical or stereotyped procedure. Certainly there are many things that teachers can do to aid their pupils in solving the kind of written problems that appear in textbooks. Perhaps the oldest method is still one of the best and that is to assign problems and to help the individual pupils to understand the language and the mathematical relationships and then to perform the necessary computations. But each pupil cannot be given private tutoring and so we must help them as groups. An experienced teacher soon recognizes the needs of her pupils and will give such help as seems warranted but she will not do the thinking for her pupils. Each one must travel this road as an individual if he is to learn to think and to use all of the sublearnings that go into thinking and problem solving.

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Division by a Two-Figure Divisor

Louis E. Ulrich, Sr.
Wm. T. Sherman School, Milwaukee

It is now generally accepted that arithmetic meanings reside in the number system in use. Long division examples are readily understood when the decimal computation process patterns required by them are recognized. The purpose of this paper is to show, that since the decimal system of computation and notation is the generally accepted system, methods and meanings must be consistent with it to get results just as the rules of baseball or any other sport designates the skills to be acquired and the procedures to be followed in order to win. If this is true then decimal computation should provide the meanings teachers and students require not only to work examples but also to solve problems.

I. Long division is the reverse of multiplication in meaning and computation.[1]

Thus:

$$\begin{array}{r} 36 \\ \times 25 \\ \hline 180 \\ 72 \\ \hline 900 \end{array} \qquad \begin{array}{r} 25 \\ 36\overline{)900} \\ 72 \\ \hline 180 \\ 180 \\ \hline \end{array}$$

Notice;

a. In multiplication we work from right to left and in division from left to right.
b. The transformation in multiplication is changing each ten smaller units to a larger decimal unit, in division the transformation is in the reverse.[2]
c. In multiplication we end with the sum (product) in division we begin with the sum (dividend).
d. The first partial product in multiplication is the last in division and the last partial product in multiplication is the first in division.
e. In multiplication the product (sum) must always be found. In division the dividend (sum) is always given. This helps us to understand the social situation.
f. Long division however is more difficult than multiplication because it includes the division facts, proficiency in subtraction and multiplication, the creation of successive partial dividends and the finding of quotient figures. Discussion should therefore follow multiplication and subtraction.
g. We see then that division cannot be explained as mere repeated subtraction. It is much more than that. In the following division examples no subtraction is required:

$$\begin{array}{r} 1.\quad 20 \\ 74\overline{)1485} \\ 148 \\ \hline 5 \end{array} \qquad \begin{array}{r} 2.\quad 100 \\ 10\overline{)1000} \\ 10 \\ \hline 00 \end{array} \qquad \begin{array}{r} 3.\quad 201 \\ 35\overline{)7035} \\ 70 \\ \hline 35 \\ 35 \\ \hline \end{array}$$

Repeated subtraction explains what is done with concrete materials and upon the abacus, but does not represent what is decimally done in abstract division on paper.

II. Are all the long division examples discrete or are they related? If related what are their process patterns so that their relatedness may be discovered. Such relatedness is very important. When examples are arranged according to their relatedness they may be taught in the order of their complexity so as to take advantage of the law

[1] Louis E. Ulrich, Sr., "Streamlining Arithmetic," Lyons & Carnahan, Chicago, 1943, p. 120–121.
[2] Louis E. Ulrich, Sr., "100% Automatic Response," Arithmetic Teacher, October 1957, p. 161.

of transfer of learning. When, however, this attempted classification is based upon superficial inspection as to whether the frequencies are large or small, contain zeros or not, or whether or not the numbers are even or odd, or when elements of operations that should have been mastered before division is attempted are included, then the number of types and their complexity scale will vary with each investigator since a constant by which to judge complexity was not employed.[3] This constant consists of the processes new to long division and are:

a. Division facts.
b. Find the successive quotient figures.
c. Building the successive partial dividends which includes bringing down.

III. METHOD

a. In order to find quotient figures the classes were taught to mentally multiplicand by one figure multiplier. As:

$$\begin{array}{r} 35 = 7\times 3 \text{ tens} = 210 \\ \times 7 \quad 7\times 5 \text{ ones} = \ 35 \end{array} \text{ together } 245.$$

Both classes also reviewed these terms: 265 means:

Frequency		Decimal Unit		Product
2	×	100	=	200
6	×	10	=	60
5	×	1	=	5
				265 sum

b. The following helps were also practiced. In this example: $36\overline{)7236}$ the frequencies of the divisor 36 are less than the first two frequencies 72, of the dividend. Therefore 72 is the first partial dividend.

c. In $36\overline{)10872}$ the frequencies of the divisor 36 are together greater than the first two frequencies of the dividend, therefore the first three frequencies, 108, of the dividend are the first partial dividend.

d. To find the trial quotient figure, divide the first or first two frequencies of the dividend by the first frequency of the divisor as the situation may require and multiply the whole divisor by it. If the result is too large try the next smaller trial quotient figure.

e. If the trial quotient is greater than 9—try 9. In any case, if the trial quotient is too large try a smaller one until the correct quotient figure is found. The quotient figure is correct when after subtracting the difference is less than the divisor.

f. When the divisor is a teen several tries may have to be made.

g. When the last frequency of the divisor is 5 or more try a quotient frequency one less than when the last frequency of the divisor is 5 or more. When the student has learned to multiply mentally, finding the real quotient figure is done rapidly by inspection.

h. Every frequency brought down together with the remainder, if any, must be divided by the whole divisor. When such division is impossible a zero must be placed in the quotient at that place.

IV. ASSUMING THAT THE PRECEDING FUNDAMENTAL OPERATIONS HAVE BEEN MASTERED we may begin by showing children that all division examples are made up of one or a combination of more than one of the following basic process patterns or types.

a. $12\overline{)24}$ = (D.M.) means divide-multiply.

b. $12\overline{)27}$ = (D.M.S.) means divide-multiply-subtract 3,898.

c. $12\overline{)108}$ = (T.T.D.M.) means transform twice-divide-multiply 629.

d. $12\overline{)110}$ = (T.T.D.M.S.) means transform twice-divide-multiply-subtract 39,538.

Incomplete end patterns are those in which the frequencies brought down together with or without a remainder are not large enough to permit another division as:

[3] Louis E. Ulrich, Sr., "A Study of the Process of Transformation," *The Mathematics Teacher*, November 1947, p. 349.

$$\begin{array}{r} 20 \\ 74\overline{)1485} \end{array} \qquad \begin{array}{r} 40 \\ 37\overline{)1492} \\ 148 \\ \hline 12 \end{array}$$

When the basic patterns have been mastered, joining them to form larger examples helps children to analyze any example and to recognize its partial dividends as:

$$\begin{array}{r} 2100908 \\ 13\overline{)27311804} \end{array} = \begin{array}{l} (1)\ \begin{array}{r} 2 = \text{(T.D.M.S.)} \\ 13\overline{)27} \\ 26 \\ \hline 1 \end{array} \qquad \text{basic type 2} \\ \\ (2)\ \begin{array}{r} 1 = \text{(T.D.M.)} \\ 13\overline{)13} \end{array} \qquad \text{basic type 1} \\ \\ (3)\ \begin{array}{r} 009 = \text{(T.T.D.M.S.)} \\ 13\overline{)118} \\ 117 \\ \hline 1 \end{array} \qquad \text{basic type 4} \\ \\ (4)\ \begin{array}{r} 08 \\ 13\overline{)104} \\ 104 \end{array} \qquad \text{basic type 3} \end{array}$$

V. PROBLEM SOLVING

In order to make division meaningful and functional, social situations such as the following were employed and children were asked to bring in written problems of their own. Let us examine the following social situations:

A. Eight girls from our troop were each given a twenty-five cent ticket for the Holiday Fair. How much was taken from the treasury to pay for the tickets?

$$\begin{array}{r} .25 \\ \times\ 8 \\ \hline \$2.00 \end{array}$$

B. Two dollars was taken from the troop's treasury to buy twenty-five cent Holiday Fair tickets, one for each girl. How many tickets were purchased? How many girls went?

$$\begin{array}{r} 8\times \\ .25\overline{)\$2.00} \end{array}$$

C. Two dollars was taken from our troop treasury to buy eight Holiday Fair tickets. How much did each ticket cost?

$$\begin{array}{r} .25 \\ 8\times\overline{)\$2.00} \end{array}$$

The mathematical principle involved here is that the sum (product, dividend) is equal to the number (multiplier, quotient) of its related like parts (addends, multiplicand, divisor). It follows that the dividend divided by the divisor (size of parts) equals the quotient, and the dividend divided by the quotient (times, number of parts) is equal to the divisor. In problem A we are to find the product of eight equal parts of twenty-five cents each. In problem B we are to find the ratio of a like part to the whole. In problem C we are to find the number of like parts equal to the whole. In finding the ratio between an equal part and the whole, as in problem B, we include all problems that require the answer: how many times, how many times larger, or how many times more. The real question asked by problem B is: How many times was a twenty-five cent ticket bought? The real question asked in problem C is: How many cents did each ticket cost? In thus finding the size of an equal, that is, fractional part, we subsume all problems that ask the question: how many, how much, how large? Thus a mathematical principle inherent in the algorism is applied to the various problem situations. The real meaning may often be hidden by the usual language employed in the problem. Computationally, before the student is able to recognize a situation requiring the solution of either of the two types of division problems he must be able to multiply and divide meaningfully. Without that ability he would not be able so much as to recognize the problem situation. It would seem then that the ability to compute meaningfully is a prerequisite to problem solving.

VI. MULTI-SENSORY AIDS

The use of tangible devices when carefully employed may aid the application of computation processes to the solution of problems. It must however be recognized that the teacher possesses the principles and concepts involved and helps the students recognize these by applying them to many prepared situations. The principles do not come out of the devices or situations but are applied to them.

The usefulness of concrete devices lies in their consistency with the decimal system. For example: to show the meaning of $25\overline{)125}$, students are sometimes told to subtract successively 25 from 125 and to count the number of subtractions to arrive at the quotient. Such a procedure is confusing for the simple reason that 25 can be subtracted from 125 but once. Succeeding subtractions are 100−25; 75−25; 50−25; 25−25. Nothing that was here done represents what is required in long division on paper. During abstract decimal division of this example we divide one unit of a hundred, two units of ten, and five units of one by twenty-five considered as ones. To do so we must transform one hundred to ten tens, and twelve tens to one hundred twenty ones. Notice also that there is no way to express a frequency greater than nine in the decimal system. Hence it is necessary to write 10 hundreds and 10 tens. To write "ten tens" would be ascribing both size and frequency to the word "ten." Here we see again that the thinking on the objective level does not describe the thinking on the abstract computative level.

To use another illustration: Suppose we wish to show a class how to divide six candy bars equally among four guests. We can readily show that each guest would get at least one whole bar. The two bars over could be placed end to end and then divided into four equal pieces. We would then have four-fourths of two pieces or 2×2 halves of one piece. Then each guest would receive one piece and one-half of one piece. On paper we could compute it in one of three ways:

a. $\begin{array}{r} 1\text{—R2} \\ 4\overline{)6} \\ 4 \\ \hline 2 \end{array}$ b. $\begin{array}{r} 1\frac{2}{4}=1\frac{1}{2} \\ 4\overline{)6} \\ 4 \\ \hline 2 \end{array}$ c. $\begin{array}{r} 1.5 \\ 4\overline{)6} \\ 4 \\ \hline 20 \\ \underline{20} \end{array}$

Only the third way is decimally correct. The first is incomplete, and the second uses common fractions together with decimal numbers. The point to emphasize is that working with the concrete did not illustrate abstract decimal computation. The very purpose of abstract computation is that it enables us to think thoughts and do things impossible on the concrete level. Hence it is doubtful that we can get abstract meanings from sensory aids. The reverse is probably true, namely, that abstract decimal concepts and processes are applied to the tangible situation.

VII. RELATIONSHIPS

With the foregoing principles as a basis we are able to recognize the following relationships:

a. Problem C above and its algorism is the transition point to fractions since

$$8\overline{)\$2.00}\ .25$$

also means $\frac{1}{8}$ of \$2.00 = \$.25.

b. The algorism of problem A shows that each multiplication fact is basic to two division facts and supplies the meanings to the problem situation as:

1. $\begin{array}{r} .25 \\ \times 8 \\ \hline 2.00 \end{array}$ 2. $\begin{array}{r} 8\times \\ .25\overline{)2.00} \\ \underline{2.00} \end{array}$

3. $8\times\overline{)2.00}\ .25$

c. That the facts 8×.25 and .25×8 are not related but apply to different social situations.

d. That the multiplication fact and its concomitant division facts make up the three cases of percentage, profit and loss, interest, commission, and insurance.

e. It also shows the three possible problem situations in common fractions as:
   1. What fraction of \$2.00 is 25¢?—finding the fraction.
   2. \$.25 is $\frac{1}{8}$ of what amount?—finding the amount or whole.
   3. $\frac{1}{8}$ of \$2.00 is how much?—finding the fractional part.

VIII. Classroom Procedure

In the classrooms the children were required to go to the board to compute and describe orally what they were doing. The purpose for doing so was that:

a. The teacher and class can recognize the nature of the error on the spot and help the child, if necessary, to correct it. Hence this really is a diagnostic test for each pupil at the board.

b. It provided practice in vocabulary meanings and in communication.

c. The whole class was constantly alerted to possible error. The interest and attention was thus general and uninterrupted.

IX. Tests

Two classes were taught and tested in conformity with the concepts, principles, and methods as herein described. Class A was composed of seventeen 5A pupils and sixteen 5B's. The 5A had already left long division and had spent about two months on common fractions when this test on long division, was given. Out of a total 357 examples, 338 or 94% were correct, and of a total of 336 examples the 5B's got 324 or 96% correct. The results achieved by this class shows that they had retained what they had learned. Class B was given the same test but also one on written problems. These were brought in by the children. Ten of them, five of each type, described above as B and C, were included. An analysis of all computation errors made show that:

a. The higher I.Q. children made very few errors.

b. Those that were made were often due to confusion. This may be explained by the fact that long division is a complex process including not only division but also processes required by addition, subtraction, and multiplication. Such a combination of all fundamental operations naturally taxes the attention span and produces confusion resulting in errors that may appear as carelessness especially when the student did not make the same errors in other examples. Types of errors made were as follows:

Confusion

```
   20                    erasures
46)1059               37)1482
   92                    erasures
  →79
```

Subtraction

```
   83
12)988
   96
  →38
   36
    2
```

Multiplication

```
   27
46)1059
  →72
   339
   322
    17
```

Quotient Figures

```
   24←
25)725
   50
   225
   220
     5
```

First Partial Dividend

```
  90          99
10)1000     10)1000
→100        →90
   0          100
              90
              10
```

Carelessness

```
  21
38)799
  76
  39
  38
→11
```

Forgot to Add the Transformed Frequency

```
   7
57)450
  349
   11
```

It appears that these children were well taught in the prerequisite fundamental operations.[4] The results achieved here were then not solely due to good teaching of long division. Errors in long division may sometimes be due to the fact that children are not prepared for it. The results seem to show that a good understanding of computation meanings helps written problem solving.

TESTS USED IN THE STUDY

Arithmetic Examples in Long Division

1. $21\overline{)42}$	2. $27\overline{)57}$	3. $15\overline{)120}$	4. $57\overline{)450}$
5. $27\overline{)543}$	6. $43\overline{)870}$	7. $12\overline{)988}$	8. $38\overline{)799}$
9. $25\overline{)725}$	10. $34\overline{)573}$	11. $74\overline{)1485}$	12. $37\overline{)1482}$
13. $58\overline{)1173}$	14. $46\overline{)1059}$	15. $99\overline{)1485}$	16. $34\overline{)2109}$
17. $10\overline{)1000}$	18. $35\overline{)7070}$	19. $46\overline{)9248}$	20. $93\overline{)9951}$
21. $75\overline{)7878}$			

Arithmetic Problems in Long Division

1. Last summer my family and I took a pleasure trip. It took us 21 hours to drive 798 miles. How many miles did we average per hour?

2. Our Scoutmaster bought 372 Scout Circus tickets. There are 31 boys in our troop. On the average how many tickets did each boy sell?

3. My mother bought a box of Tootsie Rolls for my club. In the box there were 240 little Tootsie Rolls. Sixteen girls were coming. How many Tootsie Rolls would each girl get?

4. I belong to the Real Book Club and I bought 12 books in one year. The total cost of the books was $16.50. On the average how much did each book cost?

5. Social Center has 11 club rooms. On Saturday at the Center, there were 165 children present. On the average, how many children were in each club room?

6. Our class will pay $10.00 for a bus to take us to the Museum. There are 35 children in our class. How much must each child pay?

7. This year our school collected 1013 ar-

[4] "100% Automatic Response," ARITHMETIC TEACHER, October 1957, p. 161.

ticles of canned and packaged foods for the Salvation Army. There are 22 rooms in Sherman School that contributed. On the average, how many items did each room give?

8. A pilot flew a total distance of 3705 miles in 13 hours. What was his speed per hour?

9. My mother and I went to the grocery store. They had a sale on candy bars. One box of 24 candy bars cost 87¢. How much would each candy bar cost?

10. In the 5A grade we study 288 new spelling words. There 18 words in each lesson. How many lessons do we have during the semester?

EDITOR'S NOTE. Mr. Ulrich's method has produced excellent results as shown by the tests given. The tests used have been reprinted so that other teachers may use them and compare their results with those in grade five of the Wm. T. Sherman School of Milwaukee where Mr. Ulrich is the principal. The teachers of the two groups of pupils were Miss Elvira Ankerson and Miss Arona Kumath. It is recognized that many people will not agree with the analytic approach to division which was employed. There are other approaches that produce good results. Perhaps the important point here is to note that children will learn long division if they are systematically taught and given practice in the process. Mr. Ulrich pointed out that the social and economic uses of division do not match the steps in the normal algorism of division and hence there is difficulty in establishing a "situation rationalization" of the procedures. Another point we should consider is that of individual differences in the approach to division and in the steps of learning the conventional pattern. Many teachers have come to believe that for some pupils it is a mistake to call attention to steps and sub-steps of the process. Long division is probably the hardest thing that pupils learn in the elementary school. There is ample opportunity to experiment with different methods but we do agree that a method that produces results is one that should be given serious considerations.

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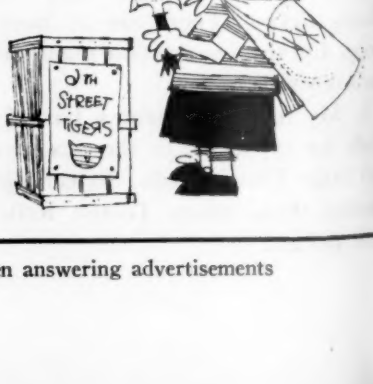

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Book Reviews

Growth in Arithmetic, One by One, Grade 1, (*Teacher's Edition*), John R. Clark, Charlotte W. Junge, Caroline Hatton Clark. Yonkers: World Book Co., 1957. Paper, iv+132 pp. Pupil's workbook, paper, 128 pp. $.69.

Growth in Arithmetic, Two by Two, Grade 2, (*Teacher's Edition*), John R. Clark, Charlotte W. Junge, Caroline Hatton Clark. Yonkers: World Book Co., 1957. Paper, vi+150 pp. Pupil's workbook, paper, 144 pp. $.69.

The above materials have several commendable features and very few to criticize for the teaching of basic arithmetic in Grades 1 and 2. In their *Teacher's Edition* the authors are explicit and brief in presenting six valid principles for learning arithmetic. These are: (1) Children must see arithmetic as a normal part of their environment and of their daily experience; (2) Learning proceeds best when children are motivated by a purpose, or desire, and when they have an active part in the learning activities; (3) Readiness is prerequisite to all learning; (4) Meaning is given to arithmetical facts and processes through use of concrete materials; (5) Practice should be for developing and extending understandings, not specifically for memorization, and (6) Arithmetic experiences should be accompanied by feelings of satisfaction and competence.

In accord with these basic assumptions, one finds a detailed outline for the full development of every lesson including Purpose, Readiness, Procedure and Supplementary Exercises. This program of readiness offers rich concrete experiences for thinking about number before the child meets each lesson in the book. Procedure for the optimum use of the book in each lesson is suggested and the supplementary exercises present activities for the rapid and the slow learner as well as interesting games for drill. Each *Teacher's Edition* has its own list of teaching aids, number games, and books for the reading table. This should be of great help to the beginning teacher. There is also ample leeway for the experienced teacher's own ingenuity. Since ideas as such cannot remain latent, they will nourish new ideas in the resourceful teacher.

All the pages of the pupil's book are reproduced in the *Teacher's Edition* as well as a series of tests for which is included a special stencil scoring key with directions. There is no charge for the excellent *Teacher's Edition* when the request accompanies a class order.

Throughout the books one finds the child's learning directed toward the ever prevailing mathematical questions: (1) How many? and (2) How much? These are presented in a variety of child-like situations with emphasis on the number system, number relationships, problem solving and skills. Herein number is mathematical and personal, foundational and social with emphasis on sensory experiences. Thus number and child develop simultaneously and hence, the title of the series, *Growth in Arithmetic*.

If some features are lacking in these books, one must search to find them. In the two approaches to number: (1) the collectional and (2) the line, it appears the line approach was somewhat minimized. Reference is made to bundles of ten and hundred, the hundred chart, the 100 bead frame and the abacus. The clothesline calendar is mentioned but where is computation with whole numbers and fractions on the ruler, the "continuous" number line below the chalk tray, and the thermometer for the second graders? Perhaps these are left for the resourceful teacher.

None-the-less the authors have presented a well-balanced program in their attractive text-workbooks in which the child should be delighted to work and proud to own.

ANN C. PETERS

Understanding Arithmetic Series, Grades 1–8 (teacher's manuals, textbooks, workbooks), E. T. McSwain, Louis E. Ulrich, Sr., Ralph J. Cooke. River Forest, Illinois: Laidlaw Brothers.

The series consists of the following types of materials: Paper-back textbooks and facsimile teachers' editions for grades one and two, cloth-bound textbooks and facsimile teachers' editions for grades three including eight, paper-back practice exercise books for grades three including eight, and semester tests for grades three including eight.

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The *Teacher's Editions* for grades three including eight are better organized and contain more helpful material than those for grades one and two. Are the first two grades not as important in building foundations for an understanding of our number system?

Some of the general features of the *Teacher's Editions* (grades 3–8) are as follows:

1. The back of each book (pages 218–224) contains a list of Arithmetical Meanings That Children Need to Understand. In the main this is a good list. Some of the ideas are ones that might be questioned, but at least it is a step in the right direction.
2. Page 217 of each book contains a few cross-number puzzles appropriate for the grade.
3. The Introductory Section (pages 3–8) of each book includes excellent material that a teacher could use as a guide for a year's work. This material is arranged under the following headings: (a) Ideas which are basic to effective instruction in arithmetic, (b) Goals and objectives of *Understanding Arithmetic*, (c) General Suggestions concerning instructional practice, and (d) Organization of the teaching plans.
4. For each lesson of the pupils' textbooks, specific instructional procedures are suggested. These usually include ideas for appropriate concrete materials, applications, provisions for individual differences, and suggestions for class discussion. The purpose of each lesson is stated, usually followed by a reference to the appropriate section on arithmetical meanings at the end of the manual. The page of the exercise book which correlates with the lesson is also listed.
5. For each chapter of the pupils' textbooks, the manuals include an outline of the chapter content, and a suggested list of concrete materials, semi-concrete materials, audio-visual aids, and references.
6. Following the lesson plans for each chapter is a chapter test (with answers) which is different from each chapter test in the pupils' textbook.

A specific feature of the teachers' editions that is good is the suggestion by the authors that the word "change" be used in place of the words "carry" and "borrow" (*Teacher's Edition*, pages 219–220).

A teaching procedure suggested in this series that is too often overlooked is the one to be used with children when they are counting objects. The following directions are given:

> Make certain that the child touches and enumerates each object in proper sequence and moves it to a position slightly closer to those objects already counted, in order to emphasize the cumulative nature of counting to find *how many*, as opposed to using numbers to designate *position* or *sequence*. (*Teacher's Edition*, Book 1, page 4)

The Practice Exercise Books have perforated pages and provide space in which the pupil may do the computation necessary for the solution of the problem. A chart for

the teacher indicates the correlation between pages in the textbooks and in the workbooks. The workbooks are designed to be used *after* the page in the textbook has been studied.

The semester tests are in separate booklet form with perforated pages and a chart on the front cover for recording the test scores for both semesters. Each semester test consists of four separate tests. One test is on understandings, one on fundamental processes, and two on problem solving. Answers for the tests are in a separate booklet.

Each chapter in the pupils' books (grades 3–8) provides a review and a self-test, and the last chapter is a review of the year's work. A few of the basic operations and principles which have been developed in a way that would lead to understanding are the multiplication of two-digit numbers, area, and the laws of fractions. But many of the approaches to new concepts are mechanical, with little or no development of understanding.

Some of the aspects of the series with which the reviewer disagrees are as follows:

1. The series is not consistent in philosophy. The teachers' manuals stress the discovery method, then the material in the pupils' textbooks is arranged in such a way as to forestall any incentive for discovery on the part of the pupil. The generalization that the teacher is leading the pupil to discover is on the page in full view, usually in darker print than the other sentences on the page, and many times has a colored background for special emphasis.
2. Students are expected to discover a generalization after having worked too few problems. For example, a lesson in Book 7 (page 215) has been prepared to help pupils learn the meaning of pi. Two problems are given for the student to work; then the generalization is stated. Is this adequate? The manual tells the teacher, "You may wish to have the pupils experiment further by measuring the distance around various circular objects and measuring the diameter." A word stronger than *wish* should be used. One cannot and should not ask students to generalize on a basis of *two* examples.
3. An inadequate statement is made about zero. In the teacher's manuals under the section entitled, "Arithmetical Meanings That Children Need to Understand," the following statement is made:

   > The arithmetical symbol, 0, called *zero*, is not a digit. It is the symbol used in the number system to denote the absence of units in one or more of the value positions.

   Is not this a limited idea of zero?
4. Excessive use is made of questions which are not thought provoking. In the pupils' books, questions are stated so as to reveal the answer, and in many cases the answer is the one word—"yes." This is particularly true for those lessons in which a new concept is being developed.
5. The "inversion method" of dividing fractions is not rationalized. The division of fractions is first developed through the use of the "common denominator" method. After several pages of working problems by this method, an illustration is given of the "inversion method" (Book 6, page 213). This is accompanied by the following description: "First the divisor was inverted and then the fractions were multiplied. Note that cancellation was used." No understanding is developed as to *why* this trick works.

The teacher's manuals (grades 3–8) are the strongest part of this series. They have some excellent ideas, but even so, one must use them with care.

JULIA ADKINS

Numbers at Work Series, Grades 1–8, David H. Patton and William E. Young. Syracuse, New York: Iroquois Publishing Company, 1954–1957.

Textbooks for grades 3–8 and workbooks for grades one and two (including primer)

were submitted for review. In the absence of teacher's manuals, it is presumed that they do not exist; if such is the case, many beginning teachers would feel handicapped and many critical teachers of experience would look in vain for statements of philosophy, and guides for extending the applications of topics presented in the texts.

The books are bound well, with colorful, eye-appealing covers. Equal attention has not been given, however, to the illustrations within the books. For the most part, they are lacking in brilliance and child appeal. The print is poor, and exercises give the impression of being crowded together.

There seems to be a lack of consistent emphasis on developing meanings. For example, the child is told to "carry the one" when adding 9 and 7 is an example involving two figure additions. He is also told that "zero is not a number," but only a place holder. Place value is not stressed as consistently as it should. There is a lack of encouragement to estimate answers. There is a shortage of mental arithmetic. In the eighth grade book there seems to be some confusion as to the meaning of the term "per cent." Whereas the term is defined as "hundredths," it is also used in the following manner in the formula "base×rate=per cent."

More careful editorial work would have improved upon sentence structure in the reading material, of which the following are examples: "We sure should be glad we are Americans"; "Our minister or priest, our doctor, our teacher, our trained nurse, our social worker, our lawyer; without them our lives would be much more difficult." Unit titles appearing in the tables of contents are not repeated on the pages where one would expect to find them. In fact many unit titles lack challenge.

Content is treated spirally to provide for periodic review and extension of concepts. There is a good supply of practice material, but most of it appears, according to page reference, some distance away from the page where initial instruction is given. Inadequate stimulation is given to the advanced pupil. There seems, however, to have been some attempt to introduce concepts and vocabulary earlier than one finds in certain other outlines of content.

HARRY C. JOHNSON

Arithmetic for Colleges (Revised Edition), Harold D. Larsen. New York: The Macmillan Company, 1958. Cloth, V+286 pp., $5.50.

For those who are acquainted with the first edition, it seems appropriate to state that Dr. Larsen has preserved the aims and philosophy of that edition in the revised edition. The purposes, therefore, of the revised edition are to provide a basis for a one-semester course in the principles and applications of elementary arithmetic. It is also a primary purpose of this publication, as stated by the author, to provide a textbook which may be used with college students who are preparing to teach in the elementary schools. Moreover, it seems that this textbook may be useful as the basis for an introductory course in general education where the concern is that all students acquire at least a basic functional knowledge of the principles and applications of elementary mathematics.

In the judgment of the writer it seems as though this textbook is designed to provide students with more than merely a review of the arithmetical processes with the usual problem solving exercises. It may be said that the efforts in preparing this textbook seem to have been directed toward the goal that through its use students will secure a new and enlarged view of the purposes and uses of arithmetic. The organization of the content is such that number as a system of notation is central in the discussions relating to each of the arithmetical processes. The descriptive material revealing analyses of the operations with the fundamental processes is seemingly designed for the purpose of guiding the students in enlarging their understanding of the significance of the system of notation in the developing and applying of arithmetic ideas. Exercises involving arithmetical operations in other bases are also attempts to assist students in

further clarifying their understandings of the Hindu-Arabic system of notation.

The proposed study of each process is developed by defining the process, examining its historical development, presenting its principles in operations, and analyzing the operations involved in using the process in terms of the functioning of the Hindu-Arabic system. The study of parts, common and decimal fractions, is developed in a manner similar to that described above. Probably very helpful to the students should be the exercises designed to reveal the relationships in common and decimal fractions through the presentation of the fractional parts of 10,000 and 1,000. The central theme in the study of percentage is the idea of the three kinds of problems. Naturally the discussions are designed to reveal the study of percentage as an extension of the study of fractional parts. While the exercises involving the concept of percentage have been increased in number, and rewritten, it is highly probable that the number should be even greater. The remaining content contains presentations of measurement including elementary geometry, approximate numbers, square root, and the introduction of the slide rule.

Generally speaking the exercises in each chapter have been revised and current data are included in the problem statements. There are many references included which should prove quite useful as supplementary reading for any course based upon this textbook. The list, however, does not contain many titles with publication dates within the past five years.

C. Banks Wilburn

Game Review

First Arithmetic Game, Edward W. Dolch. Champaign, Illinois: The Garrard Press, 1957. Price, Retail $2.50—School $1.50.

The First Arithmetic Game is a simple game made up of sturdy cards and provided with concise directions. It is played by drawing small picture cards from a center pile, counting the number of objects in the picture and then placing the small card under the correct number on a large card in front of the individual player.

The game has some arithmetical value for all beginners the first few times they play it since it provides interesting practice in counting. However, children soon learn to place the cards without counting. It would seem that the child is associating butterflies and the symbol *7* rather than associating a group of seven objects with that symbol.

Beginners observed playing the game were confused by the card showing two bunches of bananas, four in each bunch. They asked "Is it *2* or *8*?" The same was true of the card showing two pairs of skates. The slower child counted it as *4* while the more advanced child argued that it belonged under *2*.

The reviewer feels there is value in the game when a slow child plays it with a more advanced child, the slow child using the large cards with the number symbols and the other child using the cards with number words. The suggestion is given, "This time I would like to have you count each picture so you can be sure you put it under the right number." The mature child senses that he is helping the slower child and gains satisfaction from this.

Most games are subject to certain limitations and this game is no exception. Used wisely it could aid in developing number readiness. However, beginners find the game easy and interesting. Too, it serves very well to show young children that games using numbers can be fun, and fosters cooperative spirit for it is sometimes necessary for a child to give his partner the winning card.

Mildred Dick Pierce

Glen Rock Scores on a Canadian Test

CLIFFORD R. KREISMER
Glen Rock, New Jersey

INTEREST IN THE ACHIEVEMENT of our school children is at an all time high. Few would dispute that. Never before has the man on the street been so concerned, not only with his child's achievement compared with Tommy's next door, but with his country's children compared with another nation's youngsters. Nowadays it is not considered an irrelevant question when a parent at a P.T.A. meeting asks the principal how the hometown school district stacks up with the U.S.S.R.

Because answers are not always there and specific comparisons are practically nonexistent, the public schools of Glen Rock, N. J. began a series of reaserch projects intended to evaluate their children's performance on the standardized achievement tests constructed for and used by other nations.

In connection with the first phase of the study, the British Cotswold Measurement of Ability Test was administered in 1956 and interpreted by Mr. Albert F. Doremus, now principal of the Richard E. Byrd School. (*Mathematics Teacher*, Vol. L, No. 6, October, 1957.) Although the format and wording of the test was quite new to the Glen Rock students, they were able to achieve well above the median standardized score obtained by the British children.

The next most logical comparison seemed to be through the use of a well known standardized test that our own good friends and neighbors in Canada use for their pupils. Mr. Eugene J. Bradford, Glen Rock, N. J. Superintendent of Schools, pointed out that of all the nations of the world, Canada would most probably be the one that would still reflect both European and American principles of education.

"It has been said that Canadian education," explained Mr. Bradford, "has the benefits of both the European formal approach and the American continent's concept of education for all. It should prove interesting to subject our children to a test designed to measure academic growth of students who are exposed to these two educational forces."

The entire class of two hundred nineteen sixth graders was given the Arithmetic Test: Fundamental Operations for Grades 4–8 prepared and published by the Department of Educational Research, Ontario College of Education, University of Toronto. As the authors state, the test was "designed to provide a reliable measure of achievement in arithmetic computation over a wide range of grades and ages." Eighty items cover a variety of arithmetical operations, including addition, subtraction, multiplication, division, weights and measures, fractions, percentage, and decimals.

The Glen Rock children had an average score of 44.8. This compared favorably with the Ontario grade average of 32.8 achieved by their sixth grade youngsters in schools with five or more teachers. Using the revised grade norms computed in Ontario, this would place the average Canadian youth at 6.9 and the Glen Rock students at 7.9, a difference of a full grade.

Any fair comparison would, of course, con-

sider the average median intelligence quotient of the Glen Rock sixth grade classes involved with the test. This I.Q. figure proved to be 112.8. Therefore, a correction of the grade norm expectancy of 6.9 would provide a new norm of 7.8. This was exceeded by the Glen Rock children when they had an average class median of 8.0.

When one U. S. community of 13,000 allows a portion of their children to take a test designed for another nation's youngsters, it cannot be interpreted that, as a general rule, our children are superior or inferior to theirs. However, "spot checks" of this sort do point up the fact that here, in a typical well thought of school district, we can say that our children do achieve success when examined by a judge other than ourselves in an area of learning commonly accepted as a good test of academic capability. This we can be proud of, especially since we have not sacrificed our rich and varied educational program, stressing the basic areas as usual, for a return to the straight-laced narrowness of yesteryear.

EDITOR'S NOTE. It is heartening to note that our schools are doing a satisfactory job in arithmetic when compared to results in another country. The results reported by Professor Buswell in THE ARITHMETIC TEACHER for February, 1958 were not encouraging. Apparently we have schools in this country doing an excellent job in the many aspects of the elementary school curriculum and we have some schools whose record is poor. We must remember that in this country we are trying to do a number of things that most foreign countries do not attempt as a part of the school program. We can achieve not only computations in arithmetic but also a high level of thinking and understanding if we set that as our goal and work consistently toward that goal.

DOES GROWTH IN ARITHMETIC

One by One

(GRADE ONE)

Two by Two

(GRADE TWO)

By

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The National Council of Teachers of Mathematics

Minutes of the Annual Business Meeting

HOTEL CLEVELAND, CLEVELAND, OHIO
April 11, 1958

Dr. Howard F. Fehr, President, called the meeting to order at 4:10 P.M.

1. A motion was made, seconded, and passed to approve the minutes of the meeting of March 29, 1957, as printed in the *Journals*.

2. In his opening remarks Dr. Fehr expressed his appreciation to his fellow officers, the Board of Directors, and the membership in general for the fine cooperation he had received during his administration. He stated further that the Council was continuing to make fine progress in meeting its objectives, this progress being due to the excellent work of the members. In addition, Dr. Fehr made the following announcements:

(1) The first printing of the Twenty-third Yearbook (5000 copies) has been sold and a new printing authorized. This Yearbook was under the editorship of Dr. F. L. Wren. The title is *Insights into Modern Mathematics*.

(2) No Yearbook is being published during 1958; however, three Yearbooks will be published in successive years beginning with 1959.

(3) A new editor of the *Mathematics Student Journal* has been appointed to replace Dr. Max Beberman whose term expired. The new editor is Mr. W. W. Sawyer of Wesleyan University, Connecticut. A new format is being designed for the *Journal* which will enhance its attractiveness and make it easier to use and file. Dr. Beberman was complimented for the work which he did during his term as editor.

(4) The new Guidance Booklet has been completed and will soon be available. (See Part A of the President's report, page 464 of *The Mathematics Teacher:* October, 1957, and page 229 of THE ARITHMETIC TEACHER: November, 1957). The title will be *Mathematics People and A Good Life*. The National Science Foundation is sponsoring the project and will print at least one million copies which will be freely distributed.

(5) A special committee composed of Dr. Philip Peak, Dr. Henry Van Engen, Dr. F. L. Wren and Dr. Robert E. Pingry, Chairman, has been appointed to prepare a pamphlet on "The Role of the National Council of Teachers of Mathematics in Mathematics Education." The first draft has just been presented to the Board. After a few changes this pamphlet will be published (by June 1) and distributed.

(6) The Board invited Dr. Daniel B. Lloyd to prepare a manuscript on the History of the Council. The manuscript was presented at this session of the Board and has been referred to the Publications Board.

3. Dr. Houston T. Karnes, Recording Secretary, gave a brief review of the actions of the Board during the past year. This review included items which were thought to be of interest to the members. The review follows:

(1) Last year it was announced that the dues, in all probability, would be raised. This was done at the 1957 Summer Meeting. The new rates, effective October 1, 1958, are as follows:

(a) The individual membership dues will be $5.00. This includes one *Journal*. Both *Journals* may be had for $8.00.

(b) The student membership dues shall remain the same: $1.50 for one *Journal* and $2.50 for both *Journals*.

(c) The institutional subscription rates shall be $7.00 for each *Journal* or both *Journals* for $14.00.

(2) The new president will be the official representative of the Council at the International Congress in Edinburgh, Scotland, this summer. In addition two other members will represent the Council. They are Dr. Howard F. Fehr and Dr. Bruce Meserve.

(3) An addressograph system is being installed in the Central Office. This will increase the efficiency of the office to the members and to related organizations.

(4) The 1958–59 budget will not be presented here since it is published elsewhere in this issue. It is well to point out, however, that the general operation budget is $124,415.00. This figure represents an increase of some $23,000 over last year due to increased costs and an increase in membership. For the first time in several years the budget is balanced without resorting to the Publications and Reserve Account.

(5) Future Meetings:

(a) Summer, 1958, Greeley, Colorado. An excellent program has just been completed under the direction of Vice-Chairman Alice M. Hach.

(b) Christmas, 1958, New York City. The dates are December 29 and 30. This will give those in attendance an opportunity to take part in the New Year festivities around Times Square. An excellent program is being prepared by Vice-Chairman Robert E. Pingry.

(c) Spring, 1959, Dallas, Texas.

(d) Summer, 1959, University of Michigan.

(e) Spring, 1960, Buffalo, New York.

(f) Summer, 1960, Salt Lake City, Utah.

(g) Spring, 1961, Chicago, Illinois.

(6) Since financial support for the work of the Secondary School Curriculum Committee has not been found the Board agreed to advance up to $10,000 so that the Committee could begin to work on the program which has been devised. This amount will not go far, but it will enable the Committee to produce, within a year, some of the most needed aspects of its program; particularly that phase dealing with grades seven, eight and nine. There is still hope that the necessary money will be found to complete the entire study.

(7) Yearbooks:

(a) The Twenty-fourth Yearbook, under the editorship of Dr. Phillip S. Jones, is due to be published in 1959. The title is: *Central Themes and Concepts.*

(b) The Twenty-fifth Yearbook, under the editorship of Dr. F. E. Grossnickle is due to be published in 1960. The title is: *Arithmetic.*

(c) The Twenty-sixth Yearbook, under the editorship of Dr. Donovan A. Johnson, is due to be published in 1961. The title is: *Evaluation.*

(8) Dr. Myron F. Rosskoph has been appointed editor to plan and edit the March, 1959, issue of *The Bulletin* of the National Association of Secondary School Principals. This is by way of an invitation from the Association. It is the second invitation for such a participation on the part of the Council. This issue will cover significant movements in Mathematics Education.

4. Mr. M. H. Ahrendt, Executive Secretary, gave the following brief report:

(1) As of April 1, 1958, the total membership of the Council had reached 19,500. This is a major increase over last year. Miss Mary C. Rogers and her Membership Committee were complimented for the fine work accomplished.

(2) As of 4:00 P.M. (today) the total registration for this meeting was 1940. This is the largest meeting we have had.

(3) Due to the growth and increased activity of the Council, the office staff has grown from two persons in 1951 to nine members in 1958.

(4) The Council is presently housed in an office space of 800 square feet in the new NEA Building. Since these quarters are already cramped, office space of 1200 square feet is promised for the coming fall.

(5) The Secretary pointed out that in an operation of the Council's proportions errors were bound to occur: especially in the membership lists. He invited all to inform him whenever errors did occur.

(6) Since the fiscal year of the Council does not end until May 31, no attempt was made to give a complete financial report. Such a report will be published in a fall issue of the two *Journals*. The Secretary did indicate, however, that general operations would not require as much from the reserve account as was anticipated when the current budget was approved.

(7) Five new pamphlets were added to the list of Supplementary Publications during the year.

(8) The Secretary stated that the Editorial Staff of the N.E.A. had been of great help to him during the past year.

(9) In closing, the Secretary paid his compliments to the outstanding work of the State Representatives during the year.

5. Dr. Clifford Bell, Chairman of the Nominating and Election Committee, gave the following report of the recent balloting:

President: Dr. Harold P. Fawcett
Vice-President for Senior High School: Miss Ida May Bernhard
Vice-President for Elementary School: Dr. E. Glenadine Gibb
Directors: Mr. Frank B. Allen
Dr. Burton W. Jones
Dr. Bruce E. Meserve

6. Dr. Fehr introduced the new officers and Directors as declared elected by Dr. Bell.

7. Dr. Milton W. Beckmann, Chairman of the 1959 Committee on Nominations and Elections, invited the members to send in recommendations. The vacancies to be filled are: Vice-President for College, Vice-President for Junior High School, and three Directors.

8. Under the heading of New Business there was a discussion concerning an approach toward the uniformity of mathematical symbolism. Although there was no motion, several suggested that the Board take this matter under advisement.

9. In his closing remarks, Dr. Fehr again expressed his appreciation to the officers, directors, editors, committee chairmen, committee member and the memberships at large for the wonderful cooperation and help which he had received during his administration.

10. Report of the Resolutions Committee. This Committee was composed of Dr. Phillip S. Jones and Dr. H. Vernon Price, Chairman. Dr. Price gave the following report:

WHEREAS, the local committees listed in your convention program have assumed most of the responsibility for arrangements with hotels, schools, and industry, and

WHEREAS the magnitude of these tasks and the cheerful efficiency with which they have been executed are obvious to all of us in attendance at this the largest meeting in the history of the Council,

THEREFORE, be it resolved that the National Council of Teachers of Mathematics acknowledge in this open and public meeting its indebtedness and appreciation to the teachers of the Cleveland area and to the Greater Cleveland Mathematics Club in particular for their effort in making this convention a memorable success.

A motion was made and seconded to adopt the resolution presented by the Committee. The motion was passed unanimously by a standing ovation.

11. There being no further business, the meeting was duly adjourned at 4:50 P.M.

Respectfully submitted,
Houston T. Karnes
Recording Secretary

Receipts and Expenditures

FISCAL YEAR, JUNE 1, 1957–MAY 31, 1958

June 1, 1957—Total Cash Resources			$42,569.20
RECEIPTS			
Memberships with *Mathematics Teacher* subscriptions	$38,388.50		
Memberships with ARITHMETIC TEACHER subscriptions	13,828.23		
Institutional subscriptions to *Mathematics Teacher*	16,164.27		
Institutional subscriptions to ARITHMETIC TEACHER	10,808.89		
Subscriptions to *Mathematics Student Journal*	4,366.55		
Sale of advertising space in *Mathematics Teacher*	6,268.90		
Sale of advertising space in ARITHMETIC TEACHER	1,741.67		
Interest on U. S. Treasury Bonds	375.00		
Net profit from conventions	301.90		
Miscellaneous	92.29		
Sale of publications			
Yearbooks	28,120.27		
Supplementary publications, etc.	13,088.01		
Total receipts		$133,544.48	
EXPENDITURES			
Washington office	$44,512.19		
President's office	2,375.03		
Vice-Presidents' office expenses	135.00		
Mathematics Teacher	32,248.55		
ARITHMETIC TEACHER	12,242.18		
Mathematics Student Journal	3,022.93		
Committee work	3,637.03		
Travel by Board members	3,133.42		
Preparation and printing of yearbooks	11,436.84		
Preparation and printing of supplementary publications	8,077.34		
Distributing "As We See It"	1,255.08		
Storage and shipment of publications, miscellaneous	693.01		
Total expenditures		$122,768.60	
INCREASE IN CASH RESOURCES			10,775.88
May 31, 1958—Total Cash Resources			$53,345.08

The 1958 Budget

The Budget Committee for 1958 was composed of Miss Agnes Hebert, Mr. Jackson B. Adkins, and Dr. Houston T. Karnes, Chairman. Under the directive of President Howard F. Fehr, the committee met at the Central Office in Washington during the weekend of January 25, 1958. It was President Fehr's wishes that the committee work out a proposed budget in the meeting at the Central Office and circulate the proposed budget to the members of the Board at least one month prior to the annual meeting. In view of the importance of the budget, the committee feels that the method followed this year is most desirable. The committee had the fine services of the Executive Secretary, M. H. Ahrendt, and the availability of all the important records in the Central Office. The budget was discussed at the annual meeting of the Board on April 8, at Cleveland, Ohio. The budget as adopted by the Board is submitted below. There were very few changes in the report made by the Budget Committee. The adopted budget is predicated upon the new dues which go into effect October 1, 1958.

RECEIPTS		
Memberships and subscriptions	$113,040	
MSJ Subscriptions	5,000	
Advertising in Journals	6,000	
Interest on U. S. Treasury Bonds	375	
Total	$124,415	$124,415
EXPENDITURES		
Washington Office		
Executive Secretary (Salary)	$ 10,000	
Special Benefits	1,100	
Travel	1,000	
Secretarial and clerical help		
Salaries	30,000	
Special Benefits	1,000	
Office Expenses	11,000	
Membership Work	500	
Total	$ 54,600	
President's Office		
Office Expense	$ 1,500	
Travel	1,000	
Special Projects Fund	700	
Total	$ 3,200	
Vice-President		
Postage and secretarial help ($100 each)	$ 400	
Total	$ 400	
Mathematics Teacher		
Printing and mailing	$ 27,000	
Editorial and Office Expense	4,500	
Travel, Editor	500	
Total	$ 32,000	
ARITHMETIC TEACHER		
Printing and Mailing	$ 13,000	
Editorial and Office Expense	1,500	
Travel, Editor	500	
Total	$ 15,000	
Mathematics Student Journal		
Printing and Mailing	$ 3,000	
Editorial and Office Expense	1,000	
Travel, Editor	250	
Total	$ 4,250	
Other		
Affiliated Groups (Travel of Chairman to Annual Meeting and for *Newsletter*)	$ 500	
Committee Work	2,500	
Travel, Directors	4,000	
Total	$ 7,000	
Reserve	7,965	
Total	7,965	
TOTAL EXPENDITURES		$124,415

The 1958 Election Results

The Remington Rand Division of the Sperry Rand Corporation was again selected to count the votes for the various offices to be filled. The President, the Executive Secretary, and the Chairman of the Committee on Nominations and Elections each received a signed statement from the manager of the above-mentioned corporation certifying the results of this count of ballots.

The new officers elected are as follows:

PRESIDENT
Harold P. Fawcett

VICE-PRESIDENT FOR SENIOR HIGH SCHOOL
Ida May Bernhard

VICE-PRESIDENT FOR ELEMENTARY SCHOOL
E. Glenadine Gibb

DIRECTORS
Frank B. Allen, Central Region
Burton W. Jones, Southwest Region
Bruce E. Meserve, Northeast Region

Respectfully submitted,
Clifford Bell
Chairman, Committee on Nominations and Elections (1958)

Report of the Membership Committee

MARY C. ROGERS

Chairman, Membership Committee, Edison Junior High School, Westfield, N.J.

This has been a remarkably successful year in National Council membership growth. Needless to say, we are very happy about it. We are confident that all of you, our current members, share this feeling with us.

This reaction on your part was very evident last April at the Annual Meeting in Cleveland. It was your keen enthusiasm about the rapid growth of the Council in numbers and in practical services; your repeated remarks about "National Council's skyrocketing membership"; your offers of direct assistance toward further growth; . . . all this convinced us that our former goals, although exceptionally high, had not been too ambitious; that we should raise our sights still higher in our plans for future achievements.

There seemed to be an almost universal opinion at the Convention that the 25,000 total membership goal, set for us by Dr. Howard F. Fehr and the National Council Board of Directors in April 1957, could and should be reached by May 1, 1960 rather than by May 1, 1962 as originally planned. In light of past achievement, this shortening of time limit seems most reasonable and quite possible of attainment.

Our membership total as of May 1, 1958 was 19,497—an increase of 3316 during the past year. To achieve 25,000 total membership by May 1, 1960 will necessitate an average annual increase of about 2750 members.

True, there is the matter of increased dues for all but the Junior Members. You have already been informed of these increases in the January 1958 issues of the *Mathematics Teacher* and the ARITHMETIC TEACHER and through communication from our Washington office. We do not believe these increased dues will have any marked effect on membership maintenance and membership growth. Such has been the expressed opinion of many mathematics teachers throughout the country.

With current stress being laid on the *improvement of mathematics education at all levels;* with the *greatly enhanced interest of the general public in such improvement;* with the urgency for change in content and point of view in keeping with Modern Mathematics already being felt in classroom situation in all parts of the country, it naturally follows that the great majority of mathematics teachers feel a strong need for expert advice. In increasing numbers, they are looking to the National Council for this assistance. Let us seize every opportunity to interest *these teachers and their administrators* still more in the fine services which the Council has to offer.

Record of Membership Growth

In making our report to you, our committee has decided to speak of the goals as the "17,000 goal," the "19,000 goal," and the "21,000 goal" rather than designating any specific year for reaching these goals. We recognize May 1, 1960, however, as the ultimate date for reaching the 25,000 total. Although most states are well beyond the "17,000 goal" and the "19,000 goal" we have included analysis based on these goals in this report. After all the total goal originally set for April 1958 was 17,000.

In this report, our analyses use the "21,000 goal" as a maximum for comparative purposes at this time. It seems quite reasonable, however, that this maximum should be surpassed by May 1959 if we are to arrive at the 25,000 total by May 1960.

The present over-all picture shows we have already reached 93% of the 21,000 total.

31% of our states and territories have now gone beyond their quotas in this total.

23% of our states have attained 90%–99% of such goals.

This is an excellent beginning for the new year. We suggest that those, who have not reached their "21,000 quotas," work toward these quotas in the near future while the others aim at a higher goal. Later on this year, you will be informed of a suggested higher goal for all states. Does this meet with your approval?

Where Does Your State Stand on the 21,000 Goal?

ANALYSIS OF MEMBERSHIP GROWTH

MAY 1957–MAY 1958

	May 1958 Totals	21,000 Goals	% Reached
1. Alabama	177	237	75%
2. Arizona	203	84	242%
3. Arkansas	170	239	71%
4. California	1,445	1,231	117%
5. Colorado	243	252	96%
6. Connecticut	339	319	106%
7. Delaware	82	99	83%
8. District of Columbia	211	258	82%
9. Florida	528	491	108%
10. Georgia	226	260	87%

11. Idaho	27	25	108%
12. Illinois	1,513	1,663	91%
13. Indiana	544	758	72%
14. Iowa	338	414	82%
15. Kansas	377	435	87%
16. Kentucky	156	172	91%
17. Louisiana	356	386	92%
18. Maine	94	97	97%
19. Maryland	365	399	91%
20. Massachusetts	535	615	87%
21. Michigan	900	840	107%
22. Minnesota	465	561	83%
23. Mississippi	155	172	90%
24. Missouri	350	462	76%
25. Montana	97	84	115%
26. Nebraska	164	231	71%
27. Nevada	25	19	132%
28. New Hampshire	94	90	104%
29. New Jersey	743	785	95%
30. New Mexico	101	124	81%
31. New York	1,686	1,653	102%
32. North Carolina	237	357	67%
33. North Dakota	42	61	69%
34. Ohio	972	1,035	93%
35. Oklahoma	327	332	97%
36. Oregon	267	187	143%
37. Pennsylvania	1,271	1,287	99%
38. Rhode Island	86	99	87%
39. South Carolina	127	191	67%
40. South Dakota	54	50	108%
41. Tennessee	236	344	69%
42. Texas	890	941	95%
43. Utah	70	67	104%
44. Vermont	42	57	74%
45. Virginia	402	500	80%
46. Washington	323	319	101%
47. West Virginia	115	220	52%
48. Wisconsin	470	588	80%
49. Wyoming	53	50	106%
TOTALS	18,693	20,140	93%
Hawaii & U. S. Poss.	92	103	89%
Canada	321	282	114%
Foreign	391	475	82%
GRAND TOTALS	19,497	21,000	93%

The Membership Committee thanks each of you most sincerely for your enthusiastic cooperation and support at all times. Please accept our best wishes for a most successful professional year just ahead.

Pearl Bond	Faith Novinger
Marian C. Cliffe	Bess Patton
Mary Lee Foster	Mary Reed
Janet Height	M. H. Ahrendt and
Harold J. Hunt	Elizabeth Roudebush
Florence Ingham	Ex Officio members
Mildred Keiffer	Mary C. Rogers, Chairman

Minutes of the Ninth Delegate Assembly

HOTEL CLEVELAND, CLEVELAND, OHIO

April 9–12, 1958

[Edited by Elizabeth Roudebush, Seattle Public Schools, Seattle, Washington]

The Ninth Delegate Assembly for Affiliated Groups of the National Council of Teachers of Mathematics, met in the Rose Room of the Hotel Cleveland, Cleveland, Ohio, at 9:00 A.M., April 10, 1958, with Chairman Elizabeth J. Roudebush, presiding.

Dr. Howard F. Fehr, President of the National Council of Teachers of Mathematics, spoke to the group emphasizing the National Council's position in providing strong leadership in developing the modern mathematics curriculum. He pointed out the change in emphasis from rote learning to concept building, stressing that mathematics courses will contain some new mathematics. The most important change will be a new point of view toward the subject itself. The mathematicians, the mathematics educators, and the secondary school teachers have a definite responsibility in making mathematics a more meaningful subject.

The delegates and alternates were introduced by their respective regional representatives.

How We Publish Our Newsletter

A panel of four Newsletter editors discussed "How We Publish Our Newsletter." Mrs. Adeline Riefling St. Louis, Missouri, panel moderator, reviewed the main purposes and methods of producing a Newsletter. She shared her experiences as editor of the Missouri Newsletter, indicating the problems of collecting material and reproducing it.

Clarence H. Heinke, Capital University, Columbus, Ohio, editor of the Ohio Newsletter discussed the content and publication of two issues per year. Their Newsletter reviews important papers presented at mathematics conferences and announces programs for the coming meetings. The Ohio Council of Teachers of Mathematics prints about 1200 copies at a cost of from $65 to $105 per issue and uses some copies of the Newsletter to aid in getting new members.

Gertrude Hendrix, University of Illinois, Urbana, Illinois discussed problems involved in off-set printing of 800–850 copies of the Illinois Newsletter.

The cost of the four issues ranges from $75 to $120 per issue, depending on size. They are experimenting, at present, in selling advertising space to help cover the cost of publishing.

Madeline Messner, Abraham Clark High School, Roselle, New Jersey, gave a brief resume of the 14 years of publishing the New Jersey Newsletter. They have three issues, with 1500 copies costing approximately $250 per issue. Commercial advertising covers the cost of one issue. They have an Editorial Board with each level of teaching represented.

C. E. Flanagan, Wisconsin State College, Whitewater, Wisconsin reported that the Wisconsin Newsletter is published three times a year. He is assisted by an Editorial Committee of three. The secretary of the Wisconsin Mathematics Council prepares the gummed address labels. They print about 200 copies at a cost of $50 per issue.

USES OF MATHEMATICAL CONTESTS

John C. Bryan, North High School, Omaha, Nebraska introduced L. G. Yoder, East Palestine Ohio. Mr. Yoder of the Yoder Instrument Company described a local, project type of mathematical contest carried on at East Palestine. He said that we must develop better scientists and engineers if we are to survive and that mathematics teaching plays a very important part in this problem. Prizes are used to stimulate interest. Good newspaper publicity for the contest serves as a means of making mathematics more important to both students and citizens of the community.

Daniel B. Lloyd, D. C. Teachers College, Washington, D. C., discussed the 80 minute, objective-type test prepared under the auspices of the Mathematics Association of America, which is used to determine outstanding mathematics students in high schools participating in the contest.

Approximately 85,000 students competed in this contest in 1958. He emphasized the danger in carrying any type of contest to extremes.

COMMITTEE REPORTS

Curriculum

Frank B. Allen, Lyons Township High School and Junior College, La Grange, Illinois reported that the Secondary School Curriculum Committee has organized its work into ten specific areas of curriculum study. He emphasized the need for financial assistance to carry on the work of these Sub-committees. The help and encouragement of the membership is vital to the progress of the committee at this time.

Television

Joe Hooten, Florida State University, Tallahassee, Florida told of some of the obstacles that the Television Committee is facing. Although the work of the committee is in its initial stage, the following recommendations were made:

1. that a sub-committee be appointed to experiment and study the effectiveness of teaching by television
2. that a study be made of the effectiveness of television as a promotional agency for mathematics
3. that space be made available for use of this committee in the National Council journals
4. that financial help be solicited, subject to approval of the National Council.

Supplementary Publications

Henry Swain, New Trier Township High School, Winnetka, Illinois told the group that this committee had charge of all publications of the National Council of Teachers of Mathematics except the year books and the journals. The committee solicits and edits articles for publication. He asked the help of the Delegate Assembly in securing manuscripts, listing timely topics and suggesting people to write for future publications.

Membership

Mary C. Rogers, Roosevelt Junior High School, Westfield, New Jersey gave a very enthusiastic report concerning the membership growth and the future goals of the National Council of Teachers of Mathematics. The membership February 1, 1958 was 18,324 with an estimated membership of 19,500 today. Members were urged to work for continued growth in the organization.

Report from Puerto Rico

Francisco Garriga, University of Puerto Rico discussed the organization of a Council of Teachers of Mathematics in Puerto Rico. He expressed appreciation to Dr. Howard F. Fehr for his help during the time he spent in Puerto Rico. They have applied for affiliation with the National Council of Teachers of Mathematics.

Dues

M. H. Ahrendt, Executive Secretary of the National Council of Teachers of Mathematics explained the new schedule of dues and the need for the schedule.

Invitation from Dallas

A. W. Harris issued a cordial invitation to the members of the National Council to the Dallas meeting April 1–4, 1959.

Miss Roudebush expressed her appreciation to all who participated in the meeting. The meeting of the Ninth Delegate Assembly adjourned at noon.

Respectfully submitted,
Grace Arbogast,
Mildred B. Cole,
Secretaries of Delegate Assembly

The following people were chosen by their local groups as delegates and alternates to the Delegate Assembly. Where two names are listed, the first is the delegate, and the second one the alternate.

Arkansas Council of Teachers of Mathematics

Miss Dorothy Long, Arkansas Teachers College, Conway, Arkansas

California Mathematics Council

Marian C. Cliffe, 1457 Winchester Avenue, Los Angeles 1, California

John D. Hancock, Capuchino High School, San Bruno, California

John Forster, 351 North Highland Place, Alhambra, California

Colorado Council of Teachers of Mathematics
Forest Fisch, Colorado State College, Greeley, Colorado

Delaware Council of Teachers of Mathematics
Russell Dineen, Wilmington High School, Wilmington, Delaware

District of Columbia Teachers of Mathematics
Daniel Lloyd, D. C. Teachers College, Washington, D. C.
Miss Jane Hill, 3051 Harrison Street, N.W., Washington 12, D. C.

Benjamin Bannekar Mathematics Club
Mrs. Juanita S. Tolson, 723 Jefferson Street, N.W., Washington, D. C.

Florida Council of Teachers of Mathematics
Mr. J. Richard Sewell, 1017 Aloma Avenue, Winter Park, Florida

Dade County Council of Teachers of Mathematics
Michael Kambour, 2240 S. W. 80th St., Miami, Florida

Pinellas County Council of Teachers of Mathematics
Mrs. Carol S. Scott, 1190 8th Street North, St. Petersburg, Florida
Miss Ruth T. Reynolds, 701 Ft. Harrison Avenue, Clearwater, Florida

Georgia Council of Teachers of Mathematics
Miss Amabel Landsell, Tukman Junior High School, Augusta, Georgia

Illinois Council of Teachers of Mathematics
LeRoy Sachs, Cahokia-Commonfields High School, Range & Jerome Lane, E. St. Louis, Ill.
Miss Ruby Cooper, 1218 Jersey, Quincy, Illinois

Chicago Elementary Teachers Mathematics Club
Joseph J. Urbancek, 1112 Grant, Evanston, Illinois

Men's Mathematics Club of Chicago and Metropolitan Area
Brother Norbert, C.S.C., Holy Trinity High School, 1443 W. Division St., Chicago, Ill.

Women's Mathematics Club of Chicago and Vicinity
Miss Maude Bryan, 3108 Haussen Court, Chicago 18, Illinois

Indiana Council of Teachers of Mathematics
Mrs. Eleanor Guyer, Southport High School, Southport, Indiana

Gary Council of Teachers of Mathematics
Harold R. Jones, 4617 Delaware Street, Gary, Indiana
Miss Olive Leskow, 234 West 49th Avenue, Gary, Indiana

Iowa Association of Mathematics Teachers
Dr. Ross Nielsen, Laboratory School, ISTC, Cedar Falls, Iowa

Kansas Association of Teachers of Mathematics
Miss Gertrude Welch, Shawnee-Mission High School, Merriam, Kansas

Wichita Mathematics Association
Dr. Lottchen Hunter, 1706 N. Market, Wichita 4, Kansas

Louisiana Mississippi Branch of the National Council of Teachers of Mathematics
Mrs. A. H. Wehe, 6202 Goodwood Avenue, Baton Rouge, Louisiana

Mathematics Section of the Maryland State Teachers Association
Mrs. Margaret A. Bowers, Marriottsville, Maryland
Mr. Alfred E. Culley, 3710 Lochearn Drive, Baltimore 7, Maryland

Mathematics Teachers of Prince George's County
Mr. Gordon A. Patterson, Surrattsville Jr.-Sr. High School, Clinton, Maryland

Michigan Council of Teachers of Mathematics
Miss Catherine Meehan, 2772 Military, Port Huron, Michigan

Detroit Mathematics Club
Sigfrid Anderson, 14845 Mark Twain, Detroit 27, Michigan

Missouri Council of Teachers of Mathematics
Mrs. Adeline A. Riefling, 3507 Hawthorne Blvd., St. Louis 4, Missouri

Nebraska Section—National Council of Teachers of Mathematics
John Bryan, Omaha North High School, Omaha, Nebraska
David Wells, University of Nebraska, Lincoln, Nebraska

The Association of Teachers of Mathematics in New England
Jackson B. Adkins, Box 49, Exeter, New Hampshire
Prof. Henry W. Syer, Boston University School of Education, 332 Bay State Rd., Boston

Association of Mathematics Teachers of New Jersey
Mrs. Lina Walter, State Teachers College, Paterson, New Jersey
Mr. Max A. Sobel, State Teachers College, Montclair, New Jersey

Association of Teachers of Mathematics of New York State
Mrs. Dorothy Lape, 315 Washington Highway, Buffalo, New York
Mrs. Miriam Howard, Phelps Central School, Phelps, New York

Association of Teachers of Mathematics of New York City
Dr. Samuel Greitzer, Bronx High School of Science, 120 E. 184th St., Bronx 53, N.Y.

Nassau County Mathematics Teachers Association
Mrs. Florence Elder, West Hempstead Jr.-Sr. High School, West Hempstead, N.Y.

Ohio Council of Teachers of Mathematics
Miss Helen Brown, Steubenville High School, Steubenville, Ohio

Mathematics Club of Greater Cincinnati
Mrs. Helen S. Walter, 522 Overhill Lane, Cincinnati 38

Greater Cleveland Mathematics Club
Mrs. Helen J. Scheu, 7802 Wainstead Drive, Parma, Ohio
Mr. Richard Keuchle, 3573 Normandy Road, Shaker Heights 22, Ohio

Greater Toledo Council of Teachers of Mathematics
Mr. Joseph Jordan, Maumee High School, Maumee, Ohio

Oklahoma Council of Teachers of Mathematics
Mr. Coy Pruitt, Tulsa, Oklahoma

Oklahoma City Mathematics Teachers Council
Miss Norma Louise Jones, 1508 S. W. 43rd., Oklahoma City 19, Oklahoma

Oregon Council of Teachers of Mathematics
Mrs. Lyle Mary Wheeler, Astoria Public Schools, Astoria, Oregon
Pennsylvania Council of Teachers of Mathematics
Dr. Catherina A. V. Lyons, 12 S. Fremont Avenue, Pittsburgh 2, Pennsylvania
Mathematics Council of Western Pennsylvania
Earle F. Myers, University of Pittsburgh, Pittsburgh, Pennsylvania
John C. Knipp, University of Pittsburgh, Pittsburgh, Pennsylvania
Association of Teachers of Mathematics of Philadelphia and Vicinity
Miss Kathryn Clark, Girls High School, Philadelphia, Pennsylvania
Texas Council of Teachers of Mathematics
Arthur W. Harris, 4701 Cole Avenue, Dallas, Texas
Greater Dallas Mathematics Association
Arthur W. Harris, 4701 Cole Avenue, Dallas, Texas
Houston Council of Teachers of Mathematics
Mrs. Thelma Hammerling, 1502 Fairview, Houston 6, Texas
Utah Council of Teachers of Mathematics
Miss Eva Crangle, West High School, Salt Lake City, Utah
Mathematics Section, Virginia Education Association
Colonel William Mack, Douglas Freeman High School, Richmond Virginia
Arlington County Council of Teachers of Mathematics
Mrs. Mary D. Campbell, 608 N. Lincoln Street, Arlington 1, Virginia
Miss Alice R. Bolton, 1101 N. Kenilworth Street, Arlington 5, Virginia
Washington State Council of Teachers of Mathematics
Robert Willson, Eisenhower High School Yakima, Washington
Puget Sound Council of Teachers of Mathematics
Elizabeth Roudebush, 815 4th Avenue North, Seattle 9, Washington
Wisconsin Mathematics Council
Prof. C. E. Flanagan, Whitewater State College, Whitewater, Wisconsin

Game Review

The Ten Game, Edward W. Dolch. Champaign, Ill.: Garrard Press.

Second graders love card games and the "Ten Game" is very popular with them. It is instructive as well as entertaining.

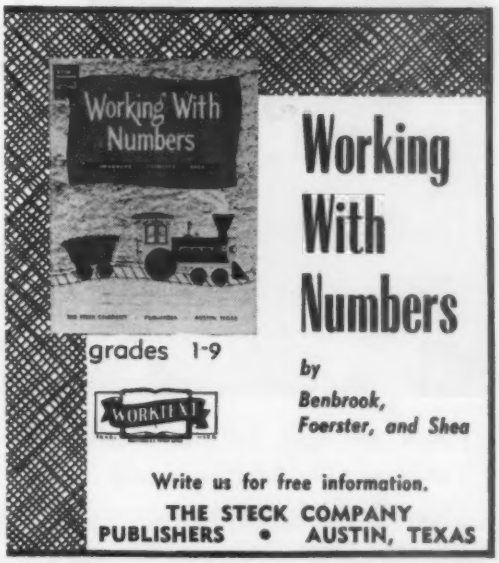
For those children who have learned the addition combinations through the use of concrete materials and symbols this game gives them much enjoyable practice in quick recognition of the two and three number combinations that make ten.

For children who are still dependent upon objects or symbols to solve problems the "Ten game" with its spot patterns on each card helps to give them experience with numbers that is fun. It provides motivation for learning the number facts more quickly.

Children who play the game at school under the direction of the teacher are soon able to play it without adult supervision and are anxious to have a game of their own to play at home.

The cards are clearly marked and easy to read but are not very durable. The sharp corners are quickly bent and cards become hard to shuffle. Plastic cards with rounded corners would be more satisfactory even though they might be more expensive.

RUTH HUTCHINSON



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